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ORIGINAL PAPER

Forecasting the Severity of Mass Public Shootings in the United States

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Abstract

Objectives Mass shootings seemingly lie outside the grasp of explanation and prediction, because they are statistical outliers—in terms of their frequency and severity—within the broader context of crime and violence. Innovative scholarship has developed procedures to estimate the future likelihood of rare catastrophic events such as earthquakes that exceed 7.0 on the Richter scale or terrorist attacks that are similar in magnitude to 9/11.

Methods Because the frequency and severity of mass public shootings follow a distribution resembling these previously studied rare catastrophic event classes, we utilized similar procedures to forecast the future severity of these incidents within the United States.

Results Using a dataset containing 156 mass public shootings that took place in the U.S. between 1976 and 2018, we forecast the future probability of attacks reaching each of a variety of severity levels in terms of the number of gunfire victims killed and wounded across three different choices of tail model, three different scenarios for future incident rates, and other parameters. Using a set of mid-range parameters, we find that the probability of an event as deadly as the 2017 massacre in Las Vegas occurring before 2040 is 35% (90% uncertainty interval [8, 72]) and we characterize how this projection varies substantially with choice of modeling parameters.

Conclusions Our results suggest an uncertain, but concerning, future risk of large-scale mass public shootings, while also illustrating how such forecasts depend on assumptions made about the tail location and other details of the severity distribution model.

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Introduction

From the 1903 mass murder carried out by Gilbert Twigg in Winfield, Kansas to the 2017 massacre in Las Vegas,[1] incidents in which victims are indiscriminately gunned down in a public place have frequently engendered widespread fear, anger and concern. Particularly troubling is the seemingly random nature of many of these attacks, targeting anyone who happens to be in the wrong place at the wrong time. Amid the onslaught of news coverage in the wake of these tragedies, the media often attempt to situate the incident within a broader context by (1) providing a "profile" of the type of individual who commits this sort of violence, (2) discussing whether the event is indicative of a larger, overall increase in this type of crime, and (3) identifying what can be done to reduce the incidence or severity of future attacks. During the late 1980s and early 1990s, a string of high-profile shooting rampages led to assertions that *mass murder* was on the rise and had become "commonplace" in the United States (Duwe 2007). More recently, massacres in places such as Orlando, San Bernardino and El Paso[2] have prompted claims that *mass shootings* have grown more prevalent and are now "routine" (Cohen et al. 2014; Korte 2016).

Even though it has been assumed that gun-related mass killings have occurred with greater regularity at various times since the 1980s, it has also been recognized that these events remain relatively rare (Duwe 2007; Fox et al. 2019). This apparent paradox—rare yet "routine"—likely reflects, in part, the outsized impact that catastrophic mass murders have on perceptions of public safety. Nevertheless, because gun-related massacres are relatively infrequent, prospectively identifying who might use a firearm to kill a large number of victims in public is exceedingly difficult, if not impossible. Fox and Levin (2011), for example, likened predicting who will commit mass murder to finding a needle in a large haystack. Many individuals fit the profile of a mass shooter, yet very few will actually turn their anger into action.

Given the infrequency and apparent randomness with which firearm-related massacres occur, it has been reasonable to assume these attacks lie beyond the realm of prediction. This also appears to be true with respect to forecasting, which focuses on the probability and severity of future events. Yet, research in other fields such as seismology, forestry, hydrology, natural disaster insurance, and terrorism has demonstrated that it is possible to develop valid estimates of the future likelihood of rare catastrophic events (Clauset and Woodard 2013). In particular, this literature has shown that such phenomena follow heavy-tailed distributions such as power laws in which most events are relatively small or low severity, while a small number of highly severe events (with sizes several times the mean) do occur. Applying models appropriate to these data to the frequency and severity of mass shootings, we attempt to forecast the probability of catastrophic events occurring in the future in the United States. Such forecasts are important for policymakers and practitioners to better understand the urgency with which continued efforts to reduce such events is

[1] Twigg committed suicide after fatally shooting 9 victims and wounding 25 more on August 13, 1903, in Winfield, Kansas. A total of 60 victims were fatally shot in the Las Vegas massacre and another 411 were injured by gunfire.

[2] There were 102 victims shot (53 fatally) in Orlando, 38 victims shot (14 fatally) in San Bernardino, and 46 victims shot (23 fatally) in El Paso.

warranted, given budget constraints. If forecasts suggest that severe mass public shootings are not likely to occur in the future, resources may be better utilized elsewhere. Moreover, the modeling and assumptions used to develop these forecasts may also advance our understanding of the underlying social mechanisms that increase or decrease the likelihood of large events.

As with other types of violence such as military conflict (Friedman 2014), terrorism (Clauset et al. 2007) or even homicide in general, the number of victims killed or injured in mass shootings is not normally distributed. For example, existing definitions have placed the fatal victim threshold for mass murder classification at either three (Dietz 1986; Holmes and Holmes 1992; Petee et al. 1997) or four (Duwe 2007; Fox and Levin 2011; Krause and Richardson 2015; Taylor 2016). Under either criterion, the vast majority of incidents classified as mass killings would have three or four fatal victims while a small (but nonzero) amount would have much larger numbers of victims (e.g., 10 or more). As such, the number of victims in mass shootings follows a heavy-tail distribution.

In this study, we use a novel strategy—at least within criminology—to estimate the risk of high-casualty mass public shootings over a variety of forecast horizons. More specifically, we use three distributions (Pareto, Weibull, and lognormal) to estimate the future probability of catastrophic mass public shootings in the United States. In doing so, we address several important questions. What is the probability that a mass public shooting as catastrophic as the Las Vegas massacre will take place? Similarly, what is the probability of an even more catastrophic mass public shooting occurring in the future?

Defining and Describing Mass Murder, Mass Shootings, and Mass Public Shootings

Given the mass confusion over the phrase "mass shooting" (Fox and Levin 2015), it is important to clarify what we mean when we use terms such as "mass murder," "mass shooting," or "mass public shooting." A mass murder is an incident in which four or more victims are killed—with any type of weapon—within a 24-h period (Duwe 2007; Fox and Levin 2011). A mass shooting is a mass murder carried out with a firearm; in other words, a mass shooting is any gun-related mass murder (Krause and Richardson 2015). A mass shooting, as we have defined it, would thus include incidents such as the 1890 Wounded Knee Massacre, the 1929 St. Valentine's Day Massacre, and recent mass murders in El Paso and Dayton.

As research has shown, however, nearly three-fourths of the mass shootings that have taken place in the United States since 1976 were either familicides or felony-related massacres (Duwe 2020). Familicides most often involve a male head of the household killing his partner (i.e., spouse, ex-spouse, or fiancée), their children, relatives, or some combination of these. Familicides almost invariably take place within the privacy of a residential setting, and the offender commits suicide in about two-thirds of these cases (Duwe 2007). In these cases, the targets are generally not viewed as random.

Felony-related massacres, on the other hand, are mass murders committed in connection with other crimes such as robbery, burglary, gang "turf wars" or contract killings (i.e., mob hits). In contrast to familicides, which are almost always carried out by a lone assailant, felony-related massacres often involve multiple offenders. While most mass killings are committed for the sake of revenge or, in some cases of familicide, out of a warped form of love (i.e., the wife and children are "better off dead"), felony-related massacres are typically more instrumental insofar as the victims are killed as a means to an end (i.e., killing eyewitnesses to a robbery seemingly offers a greater chance of evading detection). Because the violence in felony-related massacres tends to be less expressive compared to other mass killings, the perpetrators rarely commit suicide (Duwe 2004).

Compared to familicides and felony-related massacres, mass murders that involve an offender using a gun, especially an assault weapon, to shoot a relatively large number of strangers in a public location are especially newsworthy (Duwe 2000). As defined here, these mass public shootings occur in the absence of other criminal activity (e.g., robberies, drug deals, gang "turf wars", etc.) in which a gun was used to kill four or more victims at a public location within a 24-h period (Duwe et al. 2002). We also exclude from our mass public shooting classification any cases occurring in connection with military conflict or collective violence. While this definition would not classify cases such as the St. Valentine's Day Massacre or the Wounded Knee Massacre as mass public shootings, it would include incidents such as the 1966 mass murder carried out at the University of Texas in Austin, the 1991 Luby's cafeteria mass killing, the 1999 Columbine massacre, the 2007 Virginia Tech shooting and, most recently, the 2017 Las Vegas massacre. Mass public shootings can thus be seen not only as a type of mass murder, but also as a specific type of mass shooting.

Mass public shootings often dominate the news cycle because they involve, on average, a greater number of killed and injured victims than other mass murders (Duwe 2007), and the "body count" is the strongest predictor of the extent to which mass killings get reported by the news media (Duwe 2000). That mass public shooters are more likely than other mass murderers to kill strangers connotes an indiscriminate selection of victims, which increases their newsworthiness by conveying the impression that anyone could be a victim of a mass killing (Duwe 2000). Mass public shootings are also, by their very definition, highly visible acts of violence. Duwe (2000, p. 391) explains that because publicly occurring mass murders usually involve people who witnessed and survived the attack, these incidents frequently give the news media the means to "deliver a fascinating firsthand account to the audience, allowing them to vicariously experience the horror of the event." More so than other mass murders, mass public shootings tend to be exceptionally newsworthy because they are "riveting, emotionally evocative incidents" that epitomize "news as theater—a morality play involving pure, innocent victims and offenders who seemingly went 'berserk' in a public setting" (Duwe 2000, p. 391).

Mass public shootings are rare within the context of mass murder, which is itself a rare form of violence. For example, more than 1,000 mass murders have taken place in the United States since 1976, which amounts to an average of 28 per year (Duwe 2016). During the same period of time in the U.S., there have been, on average, approximately 14,200 homicides annually. Given that more than 95 percent of all homicides are single-victim incidents (Cooper and Smith 2011), mass murders make up a meager 0.2 percent of all homicides while mass public shootings account for only 0.03 percent of all homicide incidents annually.[3]

[3] These figures represent the percentage of homicides that involve 4 or more victims killed in a single incident. Of course, mass killings claim a disproportionately large number of lives as compared to most other fatal assaults. In terms of victimization, mass killings and mass public shootings in particular still account for a rather small share of all homicide victims—about one percent for mass killings and one-sixth percent for mass public shootings.

Forecasting the Severity of Rare Events

Forecasting and prediction is a core part of criminological work and becoming increasingly sophisticated. In corrections, forecasting prison populations remains a vital exercise for budgeting and resource allocation (McDonald et al. 2019). In policing, identifying future crime hot spots helps assist officers in reducing crime by being proactive (Mohler et al. 2015; Perry 2013) though some worry about the possibility that such tools may further inequities in society by distilling real-world issues down to statistical probabilities (Meijer and Wessels 2019). In other words, if data used to inform such predictions are infected with bias, the predictions will be as well (Richardson et al. 2019). If data are as comprehensive as possible, however, this seems less of a concern. Illustrating the increasing interest in crime estimation, the National Institute of Justice hosted a competition in 2017 seeking the best crime forecasting models using a variety of tools.[4] Research has indicated that the sample size of crime counts plays a major role in the accuracy of forecasting efforts (Gorr et al. 2003). Thus, the relatively infrequency of mass public shooting events seemingly poses a problem for traditional forecasting methods.

Recently, scholars have developed models using heavy tailed distributions to forecast the future size and severity of rare events such as severe earthquakes and high fatality terrorist attacks. As described by Clauset and Woodard (2013), data comprised of rare, high severity events means focusing on the upper tail of a distribution (e.g., extreme events). Research has shown that certain rare phenomena follow simple (e.g., three or fewer parameter) distributions, including city size, earthquake severity, and power outages (Clauset et al. 2009). Clauset and Woodard (2013), for example, utilized a power law distribution to arrive at a probability of 0.299 of a terroristic event the size of 9/11 (2749 deaths) over the years 1968–2007. Other distributions used by Clauset and Woodard (2013) include stretched exponential and lognormal, which provide varying estimates of the probability of such a catastrophe occurring.

Data and Method

Our data on mass public shootings were primarily drawn from the study by Duwe (2020), who relied on both the Federal Bureau of Investigation's Supplementary Homicide Reports (SHR) and news reports as data sources on mass public shootings that occurred in the U.S. between 1976 and 2018. The SHR contain incident, victim, and offender information on most murders committed in the United States. It did not become a valuable source of homicide data, however, until it underwent a major revision in 1976 (Riedel 1999). While the SHR is the most comprehensive official source of U.S. homicide data, it has several notable limitations. First, because the SHR is a voluntary program involving law enforcement agencies across the country, an estimated eight percent of all homicides are not reported (Fox 2000). Second, the SHR records frequently contain a number of coding errors (Duwe 2000; Wiersema et al. 2000). For example, Duwe (2000) found a total of 55 cases in the SHR data where victims were coded twice for the same incident, wounded victims were counted as fatalities, more than one law enforcement agency reported the same homicide, and offenders were counted as victims in murder-suicides. Finally, the SHR does not

[4] https://nij.gov/funding/pages/fy16-crime-forecasting-challenge.aspx.

include important information such as the type of location (e.g., residence, school, church, etc.) where the homicide took place or the number of victims wounded.

Compared to the SHR, news accounts usually provide more detailed information, including the location where the homicide occurred (e.g., private residence, school, workplace, etc.) and whether any victims were injured. Moreover, given that some murders are not included in the SHR, the use of news reports can help minimize the underreporting problem. Still, using news coverage as the sole source of data on mass shootings (or mass murders in general) has its own limitations, too. Even though the vast majority of mass murders, including mass shootings, are reported by the press, many receive limited, mostly local coverage (Duwe 2000; Overberg et al. 2013). Successful identification of mass public shootings that have taken place is therefore highly dependent on the news media database being used, the news organizations included within the database, and the search terms used. Indeed, not all cases are described by the news media as "mass shootings" or "mass murder," making it necessary to use expanded search terms such as "quadruple shooting," "quintuple homicide," and so on. Moreover, news coverage is generally less accessible for older incidents that occurred farther back in time.

After relying on the SHR to identify when and where gun-related mass murders (i.e., incidents in which four or more victims were killed with a gun within a 24-h period) occurred in the U.S., Duwe (2020) searched online newspaper databases to collect additional information not included in the SHR, such as the number of injured victims and the specific location where the incident took place. As a result of using this triangulated data collection strategy, which was also adopted by *USA Today* (Overberg et al. 2013) and the Congressional Research Service (Krause and Richardson 2015), Duwe (2020) was able to correct errors in the SHR data while also identifying cases that were either not reported to the SHR or were unlikely to be captured through sole reliance on news coverage. In addition, Duwe (2020) consulted unpublished mass shooting datasets from Brot (2016) and the Congressional Research Service (2014), which added a handful of cases to his dataset. Defining mass public shootings as gun-related mass murders that took place at a public location in the absence of other criminal activity (e.g., robberies, drug deals, gang "turf wars", etc.), military conflict, or collective violence, Duwe (2020) identified 158 cases that occurred in the U.S. between 1976 and 2018.

To help ensure we captured every mass public shooting that took place in the U.S. during this 43-year period, we also examined publicly available datasets such as those published by Louis Klarevas (Klarevas et al. 2019); *USA* Today (2018); *Washington Post* (Berkowitz and Alcantara 2019); Stanford University (2020); *Mother* Jones (2020); Everytown for Gun Safety (2020); and FBI Active Shooter Events (Federal Bureau of Investigation 2020). Moreover, we conducted a consensus review to determine whether cases qualified as a mass public shooting by our operational definition. More specifically, three of the authors for this study reviewed whether cases met the following criteria: (1) at least four of all victims were killed by gunfire; (2) at least four of the victims were killed in a public place or else at least half of all fatalities occurred in a public place; (3) the shooting did not occur in a private residence, although those that occurred in a non-private residence (e.g., group home or motel) were retained; and (4) the attack occurred in the absence of other criminal activity, military conflict, or collective violence.[5] If all three authors agreed

[5] While familicides almost invariably occur in a private residence, the following three cases in our dataset involved offenders who killed their family members with a gun in a public location: (1) Elyria, Ohio in 1982, (2) Oakley, Idaho in 1984, and (3) Harrodsburg, Kentucky in 1991.

these criteria had been satisfied, the incident was included in this study as a mass public shooting. If there was any disagreement, the coders discussed the case until they reached agreement on the classification.

For each case, the coders classified the incident as "yes," "no," or "maybe." Of the 188 possible cases identified, all three coders agreed on the classification being "yes" or being "no" for 175 (93.1%) of the cases. In an additional three cases, two coders agreed on the classification and the third was not sure. There was disagreement or uncertainty for 10 cases. The inter-rater reliability was assessed using Fleiss kappa, an extension of Cohen's kappa for more than two raters (Fleiss 1971). Fleiss kappa was 0.82, which indicates very good agreement between coders (Altman 1999). Overall, our mass public shooting dataset contains 156 incidents occurring between 1976 and 2018 that involved 2,360 victims who were shot, of whom 1,092 were killed (please see "Appendix A" for a list of the 156 cases).

Forecast Parameters

We estimated the probability of future catastrophic mass shootings over multiple forecast windows. In particular, we developed forecasts over periods of 5 years, 10 years, and 20 years. We constructed estimates using several different thresholds measuring the severity of the attack. Along with using the specific number of victims who were killed (60) and shot (471) in the Las Vegas massacre, we estimated the future likelihood of more catastrophic mass public shootings with larger numbers of victims killed (75 and 100) and shot (500 and 1,000). Finally, we developed forecasts based on several different assumptions about future trends in the prevalence of mass public shootings. Following Clauset and Woodard (2013), we created three sets of forecasts in which we assumed the future rate of mass shootings would be consistent with (1) average historical rates ("status quo"), (2) higher historical rates ("pessimistic"), and (3) lower historical rates ("optimistic").

To determine the future trend assumptions for mass public shootings, we analyzed data on the frequency and severity (victims killed and total victims shot) of these incidents over the 1976–2018 period. Given that the U.S. population was more than 100 million higher at the end of our study period (326 million in 2018) than it was at the beginning (214 million in 1976), we present the incidence and severity data on a per capita basis. Rather than using the conventional per 100,000 rate, we use a rate of 100 million due to the rarity of mass public shootings. In doing so, our forecasts assume that population size has an influence on the incidence of mass public shootings.[6] We model the projected incidence rate as a function of severity in two separate components: (1) prevalence over time and (2) severity distribution. Because the first component is expected to be population dependent,

[6] While some may question this assumption, there is a temporal and spatial relationship between population size and the prevalence of mass public shootings. The longer-term historical evidence has shown that the frequency of mass public shootings has, along with the size of the U.S. population, increased since the beginning of the twentieth century (Duwe 2016). In addition, the data presented in "Appendix A" reveal that roughly one-third of the mass public shootings have taken place in California, Florida, and Texas, three of the most populous states that account for 27 percent of the U.S. population (U.S. Census Bureau 2020). Further, in a recent study that examined the impact of state gun laws on mass public shootings, Siegel et al. (2020) found that state population size had a significant, positive effect on both the incidence and severity of shootings during the 1976–2018 period. More broadly, our assumption that the incidence of mass public shootings scales with population size also aligns with the long-held recognition across a wide variety of disciplines that per capita measures of social phenomena, including crime, are needed to account for the influence of population size.

we model it on a per capita basis and use Census projections. Given that the second component is not expected to be population dependent, we do not model it as such. Consistent with the different forecast windows we use, we also present the data in terms of 5-year, 10-year, and 20-year moving averages.

Trends in the Prevalence and Severity of Mass Public Shootings

As shown in Table 1, 156 mass public shootings occurred between 1976 and 2018, which amounts to an average of nearly 4 mass shooting events per year and an annual rate of 1.30 mass shooting events per 100 million. For each incidence and severity measure, we bolded the highest value and bolded and underlined the lowest value. The "N" and "Rate" values were bolded and underlined for 1979 because it was the only year during the 43-year period in which a mass public shooting did not take place. On the other hand, the "N" and "Rate" values were bolded for 2018 because it had the most incidents (10) and the highest annual rate (3.06).

As reflected in Fig. 1, the trend data show the 1986–1990 period had the lowest five-year average (0.73). Meanwhile, the 1978–1987 period had the smallest ten-year average (0.95). The highest rates, on the other hand, have generally been observed more recently. For example, the 2006–2010 period had the highest five-year average (1.77), whereas the most recent 10- and 20-year periods have had the highest average rates.

The trend data for the number of victims killed and shot further indicate that mass public shootings have recently increased in severity. Due mainly to the Las Vegas massacre 2017 not only had the largest total number of victims killed (108) and shot (563), but it also had the highest rate of victims killed (33.22) and shot (173.15) per 100 million. Similarly, the most recent five-year (2014–2018), ten-year (2009–2018), and twenty-year (1999–2018) periods had the highest average rates for victims killed and shot.

Forecast Assumptions

In Table 2, we present the assumptions used in forecasting trends in the prevalence of mass public shootings. As noted above, we developed forecasts across multiple time windows for three different scenarios—status quo, pessimistic, and optimistic. To generate the anticipated number of incidents for each forecast, we relied on the moving averages presented in Table 1 as our minimum (i.e., optimistic), mean (i.e., status quo), and maximum (pessimistic) assumptions. As such, our assumptions about future trends in the prevalence of mass public shootings are empirically grounded in the historical data.

To convert the rate averages from Table 1 into the anticipated number of incidents for each forecast horizon, we relied on a U.S. Census Bureau projection of the total U.S. population from 2019 to 2060. For our purposes, however, we focused only on the annual projections through 2039. These data indicate the U.S. population is expected to increase from 326 million in 2018 to about 372 million by the end of 2039. We then calculated the anticipated future number of cases each year based on the projected size of the U.S. population and the average rates (per 100 million) shown earlier in Table 1.

To illustrate, consider the assumption used for the ten-year "status quo" forecast for mass shootings. The ten-year moving average rate for the 1976–2018 period was 1.30. We can apply this rate to the projected size of the U.S. population for each year during the 2019–2028 period. For example, applying a rate of 1.30 (per 100 million) to a projected U.S. population of 331 million in 2019 results in 4.30 incidents. For 2024, the same rate

Table 1 Trends in the incidence and severity of mass public shootings, 1976–2018

Year	Incidence					Victims killed					Victims shot				
	N	Rate	5-Yr	10-Yr	20-Yr	N	Rate	5-Yr	10-Yr	20-Yr	N	Rate	5-Yr	10-Yr	20-Yr
1976	1	0.47				7	3.26				9	4.19			
1977	3	1.39				18	8.32				24	11.09			
1978	1	0.46				4	1.83				4	1.83			
1979	**0**	**0.00**				**0**	**0.00**				**0**	**0.00**			
1980	4	1.78	0.82			18	7.99	4.28			33	14.64	**6.35**		
1981	2	0.87	0.90			9	3.93	4.41			31	13.53	8.22		
1982	5	2.16	1.05			30	12.96	5.34			37	15.98	9.20		
1983	2	0.85	1.13			12	5.13	6.00			14	5.98	10.03		
1984	5	2.12	1.56			42	17.78	9.56			66	27.95	15.62		
1985	1	0.42	1.28	1.05		4	1.68	8.29	6.29		5	2.09	13.11	9.73	
1986	1	0.42	1.19	1.05		14	5.83	8.68	6.54		20	8.33	12.07	10.14	
1987	1	0.41	0.84	**0.95**		6	2.47	6.58	5.96		16	6.57	10.19	**9.69**	
1988	4	1.63	1.00	1.07		19	7.73	7.10	6.55		31	12.61	11.51	10.77	
1989	2	0.81	0.74	1.15		13	5.24	4.59	7.07		56	22.56	10.43	13.03	
1990	1	0.40	**0.73**	1.01		9	3.62	4.98	6.64		13	5.23	11.06	12.08	
1991	5	1.98	1.05	1.12		40	15.86	6.98	7.83		74	29.34	15.26	13.66	
1992	3	1.18	1.20	1.02		14	5.49	7.59	7.08		25	9.80	15.91	13.05	
1993	7	2.71	1.42	1.21		35	13.57	8.76	7.93		73	28.30	19.05	15.28	
1994	1	0.38	1.33	1.03		4	1.54	8.02	6.30		27	10.37	16.61	13.52	
1995	3	1.14	1.48	1.11	**1.08**	14	5.33	8.36	6.67	6.48	18	6.85	16.93	14.00	11.86
1996	2	0.75	1.23	1.14	1.09	10	3.77	5.94	6.46	6.50	14	5.28	12.12	13.69	11.92
1997	3	1.12	1.22	1.21	1.08	12	4.48	5.74	6.66	6.31	21	7.85	11.73	13.82	**11.76**
1998	3	1.11	0.90	1.16	1.11	13	4.81	**3.99**	6.37	6.46	48	17.76	9.62	14.33	12.55
1999	7	2.57	1.34	1.34	1.24	49	17.97	7.27	7.64	7.36	102	37.41	15.03	15.82	14.42
2000	3	1.07	1.32	1.40	1.21	17	6.04	7.41	7.89	7.26	18	6.40	14.94	15.94	14.01

Table 1 (continued)

Year	Incidence					Victims killed					Victims shot				
	N	Rate	5-Yr	10-Yr	20-Yr	N	Rate	5-Yr	10-Yr	20-Yr	N	Rate	5-Yr	10-Yr	20-Yr
2001	4	1.40	1.45	1.34	1.23	17	5.96	7.85	6.90	7.36	26	9.11	15.70	13.91	13.79
2002	1	0.35	1.30	1.26	1.14	4	1.39	7.23	6.49	6.78	6	2.08	14.55	13.14	13.09
2003	4	1.38	1.35	1.13	1.17	20	6.88	7.65	**5.82**	6.87	29	9.97	12.99	11.31	13.29
2004	3	1.02	1.04	1.19	1.11	15	5.11	5.07	6.17	**6.24**	26	8.85	7.28	11.16	12.34
2005	4	1.35	1.10	1.21	1.16	24	8.09	5.49	6.45	6.56	35	11.80	8.37	11.65	12.82
2006	5	1.67	1.15	1.30	1.22	27	9.02	6.10	6.97	6.72	36	12.02	8.95	12.33	13.01
2007	6	1.99	1.48	1.39	1.30	57	18.90	9.60	8.42	7.54	87	28.84	14.30	14.43	14.12
2008	5	1.64	1.53	1.44	1.30	26	8.55	9.93	8.79	7.58	53	17.43	15.79	14.39	14.36
2009	6	1.95	1.72	1.38	1.36	46	14.98	11.91	8.49	8.07	84	27.36	19.49	13.39	14.60
2010	5	1.62	**1.77**	1.44	1.42	24	7.76	11.84	8.66	8.27	36	11.64	19.46	13.91	14.92
2011	4	1.28	1.70	1.43	1.38	23	7.38	11.51	8.81	7.85	49	15.73	20.20	14.57	14.24
2012	7	2.23	1.75	1.61	1.44	67	21.35	12.00	10.80	8.64	148	47.15	23.86	19.08	16.11
2013	4	1.27	1.67	1.60	1.36	27	8.54	12.00	10.97	8.39	33	10.44	22.46	19.13	15.22
2014	3	0.94	1.47	1.59	1.39	14	4.40	9.89	10.90	8.54	33	10.36	19.06	19.28	15.22
2015	4	1.25	1.39	1.58	1.40	37	11.54	10.64	11.24	8.85	73	22.76	21.29	20.37	16.01
2016	4	1.24	1.38	1.54	1.42	65	20.12	13.19	12.35	9.66	131	40.55	26.25	23.23	17.78
2017	7	2.15	1.37	1.56	1.47	**108**	**33.22**	15.56	13.78	11.10	**563**	**173.15**	51.45	37.66	26.04
2018	**10**	**3.06**	1.73	**1.70**	**1.57**	78	23.84	**18.62**	**15.31**	**12.05**	133	40.65	**57.50**	**39.98**	**27.19**
Total	156					1092					2360				
Avg	3.67	1.30	1.28	1.29	1.28	25.35	8.92	8.36	8.15	7.81	54.88	18.88	16.51	15.63	15.03

Rates are per 100 million of the U.S. population; the lowest values for each incidence and severity measure have been bolder and underlined whereas the highest values have been bolded

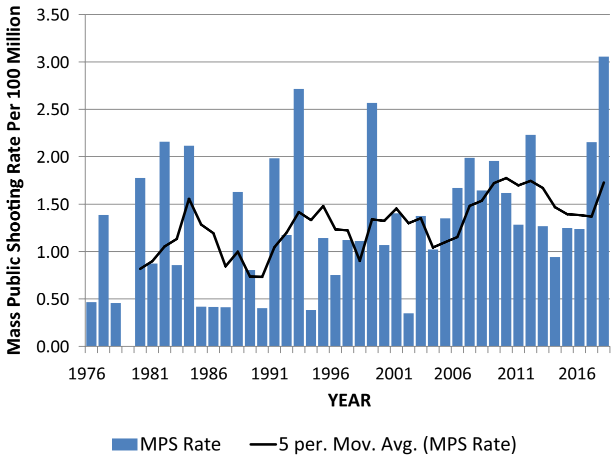

Fig. 1 Mass Public Shooting Incident Rate Per 100 Million, 1976–2018. Annual rates of mass public shooting incidents from 1976 to 2018 are based on 100 million of the U.S. population. To better illustrate the trend over time, the black line represents a five-year moving average

Table 2 Forecast assumptions

Forecasts	Five year	Ten year	Twenty year
Status Quo			
Rate	1.28	1.29	1.28
Number of incidents	21	43	89
Pessimistic			
Rate	1.77	1.70	1.57
Number of Incidents	29	56	109
Optimistic			
Rate	0.73	0.95	1.08
Number of Incidents	13	34	77

Rates are per 100 million of the U.S. population

yields 4.48 incidents due to the anticipated increase in the U.S. population (projected to be 344,814,000 in 2024). Summing the annual incident values across the 2019–2028 period produces a total of 43 mass public shootings, which was the frequency we assumed for the ten-year "status quo" forecast. We repeated this same process for the other eight forecast scenarios.

Following Clauset and Woodard (2013), we implemented three variations of distribution-fitting models to assess the tail probabilities for the severity of mass public shootings: the Pareto (power law), the lognormal, and the Weibull (stretched exponential) distributions. We fit separate models for each of two different outcome variables, the number of victims killed by gunfire in each mass public shooting ("Killed Gunfire") and the total number of victims shot ("Total Shot"). We implement each model in a Bayesian framework using the python interface (Stan Development Team 2018) to the probabilistic programming language Stan (Carpenter et al. 2017), employing Hamiltonian Monte Carlo (HMC) sampling for posterior estimation. While we expect that using a frequentist model based on a maximum likelihood estimator with bootstrap confidence estimates (as in Clauset and

Woodard 2013) would yield similar results, we prefer the Bayesian approach because it aligns to a principled analytic workflow for probabilistic modeling (see e.g. Betancourt 2018) and incorporates explicitly asserted statistical regularization through prior distributions (see e.g. Gelman and Hennig 2017). Both Bayesian and frequentist methods are widely deployed in extreme value research (see e.g. Scarrott and MacDonald 2012 for a review). In general, regularization is important in small data regimes and in bootstrapping procedures it is often necessary to incorporate simulated noise or other regularization (Raviv and Intrator 1996; Gelman and Vehtari 2014). While our results are not sensitive to the choice of prior distributions, we apply weakly informative prior information on the free parameters of the distributions as discussed for the analysis of mass public shootings by Sanders and Lei (2018). Following Clauset and Woodard (2013), we employ a discrete version of the Pareto distribution and continuous versions of the lognormal and Weibull distributions. We discuss this implementation further in "Appendix B".

We further investigated the robustness of the model distributions to the choice of the tail location (x_{min}), the minimum severity cutoff used in distribution modeling. The primary application of the fitted models is to represent the high-severity tail of the distributions; the fit to the low-severity end (the bulk) of the distribution is incidental for that purpose. In the Bayesian framework, estimation of the cutoff value for separating the bulk of the distribution from the tail is often performed using a mixture modeling approach that incorporates data throughout the support of the distribution (Scarrott and MacDonald 2012). In the case of our analysis, historical data on public shooting incidents with fewer than 4 victims is not uniformly available, and so we prefer a fixed threshold analysis. Additionally, because the total number of events is relatively low, the sample size of modeled events and the resulting uncertainty in the modeled distribution may depend sensitively on the choice of cutoff. We fit models with minimum cutoffs of 4 and 10 victims (for both Killed Gunfire and Total Shot variables), corresponding to the minimum value for mass public shootings, and the fatalities cutoff for terrorist events used by Clauset and Woodard (2013), respectively. We note that the Killed Gunfire and Total Shot variables are highly correlated, though we model their distributions independently.

Overall, we fit 108 independent models (three distributions, two variables, three scenarios, three time windows, and two minimum severity cutoffs). For each model, we make predictions of the likelihood for an event to surpass each of four different severity thresholds, t_s. For the Total Shot variable, these four thresholds are chosen at $t_s = [100, 250, 500, 1000]$. In doing so, we select several thresholds that approximate those found in the worst mass public shootings (102 in Orlando and 471 in Las Vegas) along with one (1000) that would be unprecedented in size. For Killed Gunfire, we select $t_s = [49, 60, 75, 100]$ as thresholds, with the first two (49 and 60) representing the two deadliest mass public shootings within our dataset. In using these thresholds, we not only evaluate the likelihood that a mass public shooting as catastrophic as Orlando or Las Vegas will take place at some point over the next two decades, but also the probability of an attack that would be roughly twice as severe as either incident and, thus, unparalleled in American history.

We forecast the event probability P_f using a stochastic simulation taking into account future population growth, but assuming the per capita rates from Table 2 remain constant. We apply the annual US Census population projections from 2019 through 2039 to estimate the total US population, C_y, at each year, y. For each year for each of 8000 posterior samples obtained from the model fit, we simulate a total number of events, N_e, consistent with the projected rate from a Poisson distribution. We then integrate the probability density function of the fitted tail distribution of fatalities, $P_m(x)$, from the threshold value to infinity to estimate the probability that each mass public shooting event may exceed the threshold severity. Note that for models fit with tail location $x_{\min}$ greater than the minimum value of the observational data (4), it is necessary to adjust the integral by a factor C_t equal to the probability that each simulated event in N_e belongs to the extreme $x > x_{\min}$ tail, which we estimate as a constant by integrating over the left side of the observed data distribution, P_{data}. Note that when the tail location equals the minimum value of 4, no adjustment is needed, i.e. $C_t(x_{\min} = 4) = 1$. The total probability of an exceeding event in that year, $P_f(t_s, y)$, is then calculated by compounding[7] these integrated probabilities. Finally, we combine the compound probability across all the simulated years to estimate the total cumulative probability of meeting the threshold over the time range from 2019 to 2039, $P(t_s)$. Thus, for each tail model $P_m(x)$ corresponding to a distinct choice of scenario, variable, time window, and HMC step, we perform the following calculation:

$$P(t_s) = 1 - \prod_{y=2019}^{2039} \left[1 - P_f(t_s, y)\right], \text{ and}$$

$$P_f(t_s, y) = 1 - \prod_{N=1}^{N_y} C_{\text{t}}(x_{\min}) \left[1 - \int_{t_s}^{inf} P_m(x) dx\right],$$

$$\text{where } N_y \sim \text{Poisson}(C_y), \text{ and}$$

$C_{\text{t}}(x_{\min}) = 1 - \int_4^{x_{\min}} P_{\text{data}}(x) dx$ After examining fit results at tail locations of 4 and 10 victims for each variable (i.e., cutoffs or 4 or 10 Killed Gunfire or 4 or 10 Total Shot), we selected the tail location of 10 and focused on projections associated with this value in the following Results section. Clauset and Woodard (2013) used an empirical minimization approach based on the Kolmogorov–Smirnov (KS) statistic to identify the x_{min} parameter algorithmically. Such an approach is not consistent with the Bayesian methodology we employ. As discussed previously, because modifying the value of x_{min} truncates the data distribution and invalidates comparisons between the likelihood model across x_{min} values, the nearest Bayesian alternative to the KS minimization routine would be to fix a mixture model. As we will demonstrate in the following section, we select the tail location value 10 because it generally achieves better model performance among the most severe events, which our forecasts address. We note that setting the tail location to 10 effectively discards 88% of cases for the Killed Gunfire variable and 63% of cases for the Total Shot variable in focusing the model further in the tail of the distribution. We report full results and additional comparisons between the choices of tail location in "Appendix C".

Results

We present the fitted distribution models for the tail location of 10 for the Killed Gunfire and Total Shot variables in Fig. 2. The blue shaded region shows the complementary cumulative distribution of observed event severities from 1976 to 2018. The solid lines indicate the best fit (posterior median) models for each choice of distribution. Each best fit line is surrounded by a shaded region representing the 90% posterior interval of each model fit. The shaded region grows as a function of severity because the data in that regime are more sparse and, thus, less constraining on the model.

[7] For clarity of the following equations, we note that we express the compound probability as follows: the total probability P of at least one incidence of an event over n = {1…N} trials each with probability p is $P = 1 - \prod_n (1 - p_n)$

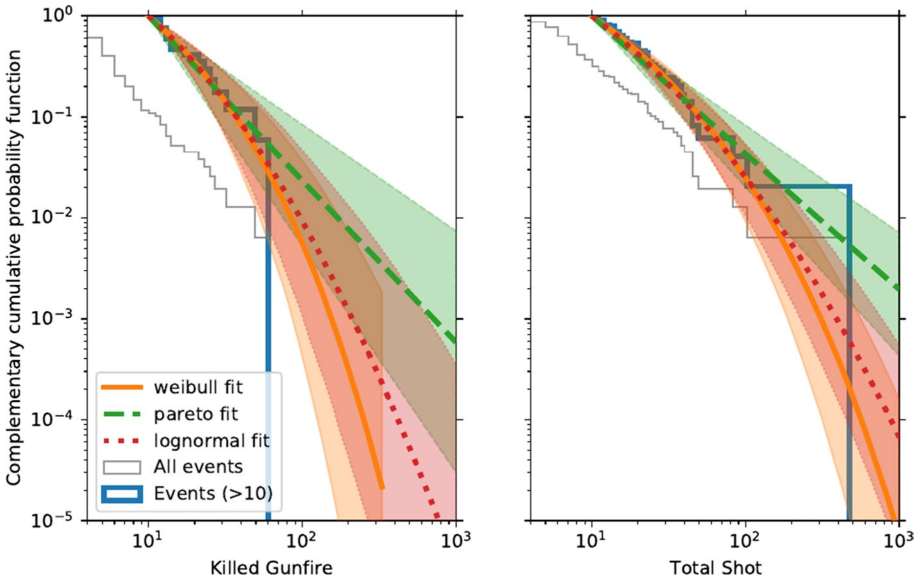

Fig. 2 Best Fit Severity Distributions, tail location of 10. Probabilistic model fits to the observed severity distribution (blue bars) of Killed Gunfire (left) and Total Shot (right). The complementary cumulative distribution functions of three different models, each fit with a tail location of 10, are shown (colored lines) together with their 90% posterior intervals (shaded regions)

A few deviations between the observed data and model fit, and between the models themselves, can be observed in Fig. 2. For the Killed Gunfire variable, the lognormal and Weibull models have relatively weak tails, under-predicting the observed rates of events with more than 20 shooting fatalities. Because such a small number of events have been observed in this regime, their true rate is highly uncertain and they have relatively little constraining power on the models; the observed distribution is consistent with the posterior interval of all models in this regime. In contrast, the Pareto model has a relatively heavy tail, better matching the observed rate of events with more than 20 shooting fatalities. The behavior for the Total Shot variable is similar, with the lognormal and Weibull models under-predicting the observed density of the very small number of events with more than 100 victims shot. Even the Pareto model predicts a lower rate of events with more than 400 shooting victims than the singular occurrence of the Las Vegas shooting would imply. These behaviors propagate to the forward projections reported below.

Figure 3 suggests that there is no overwhelming empirical basis for the preference of one model over another, but supports the selection of the tail location of 10. We evaluated the goodness of fit performance of the three different models by estimating the expected log pointwise predictive density (ELPD) goodness of fit statistic via approximate leave-one-out cross validation using Pareto-smoothed importance sampling (Vehtari et al. 2017). The use of a Pareto-smoothed importance sampling method is incidental to the choice of tail models. In order to evaluate performance in predicting the rate of the highest severity events, we estimate the ELPD only among observations with a severity greater than or equal to 10. This subselection also allows us to compare ELPD values consistently across choices of tail location without the complexity of data truncation (see the further discussion in "Appendix C"). Figure 3 displays a normal approximation of the estimated ELPD value for each model

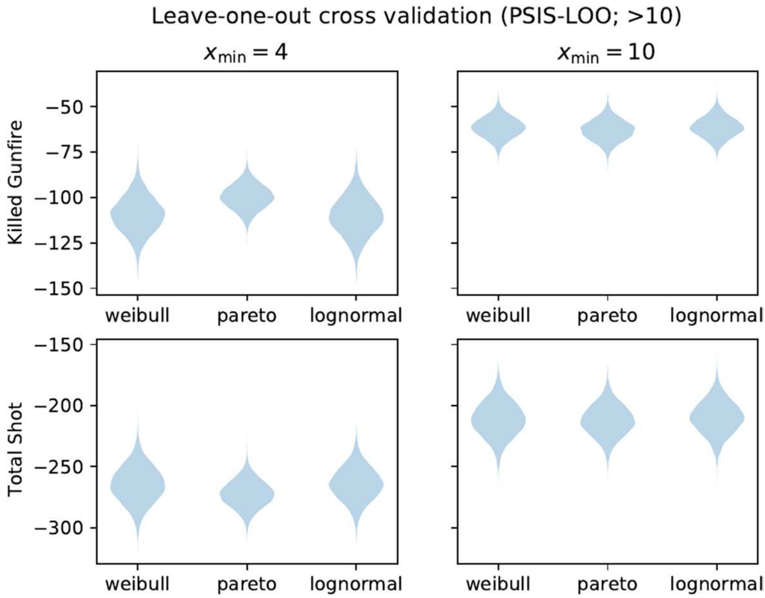

Fig. 3 Model evaluation statistics. Model performance estimates compared across all fitted models, evaluated over the x > = 10 tail of the data distribution. A higher ELPD statistic reflects a better fit to the data

and variable combination for each tail location as a violin plot; higher (more positive) ELPD scores correspond to greater model predictive accuracy. The violin plot displays the probability at each ELPD value using variation in the horizontal width of the shaded region. The figure illustrates clear separation between equivalent models with different tail locations, with the version with $x_{min} = 10$ consistently achieving higher much ELPD. For a given tail location, the ELPD measures substantially overlap across models and do not strongly discriminate between them. For models with a tail location of 10, the performance statistics are virtually identical, with no one model ever being preferred by ELPD over another in more than 57% of simulations. For models with tail location of 4, the Pareto model is somewhat favored for the Killed Gunfire variable (with highest ELPD in 80% of simulations) and somewhat disfavored for Total Shot (with lower ELPD in 67% of simulations). In all cases, the lognormal and Weibull models have essentially identical performance.

The reason why there are evident differences between the modeled probability distributions (Fig. 2) despite the fact that they cannot be distinguished with goodness of fit statistics (Fig. 3) is because of the sparse sampling at high severities. The models deviate from each other predominantly in their prediction of rates at high severity; that is, x values where there are no or very few historical examples. Because of their sparsity in this regime, the data have very little constraining power on model differences there. Together, Figs. 2 and 3 illustrate that the choice of tail model corresponds to an assumption about how to extrapolate severity rates into this extreme value domain.

Table 3 summarizes the projections for the cumulative probability of severe Killed Gunfire events for models with a tail location of 10. Similar tables for the single-year event probability and for other configurations of the tail location and modeled variable are presented in Appendices C (tail location of 4) and D (tail location of 10).

Table 3 reports the cumulative forecasted probability for any event to exceed each severity threshold at least once in the next 20 years, 2019–2039. Each column represents

Table 3 Event probability projections: killed gunfire, cumulative, tail location of 4

Scenario	Model	Window	$P_{2019\text{–}2039}$ $(x > 49)$	$P_{2019\text{–}2039}$ $(x > 60)$	$P_{2019\text{–}2039}$ $(x > 75)$	$P_{2019\text{–}2039}$ $(x > 100)$
Optimistic	Lognormal	5	33 [9.4, 65]	23 [4.6, 54]	15 [1.9, 41]	8.5 [0.54, 28]
Optimistic	Lognormal	10	39 [12, 73]	28 [5.6, 61]	18 [2.4, 48]	10 [0.68, 33]
Optimistic	Lognormal	20	41 [13, 76]	29 [6.1, 64]	20 [2.6, 51]	11 [0.75, 35]
Optimistic	Pareto	5	40 [15, 71]	32 [9.6, 62]	25 [5.9, 54]	18 [3.1, 43]
Optimistic	Pareto	10	47 [18, 78]	38 [12, 70]	29 [7.3, 62]	21 [3.9, 50]
Optimistic	Pareto	20	49 [19, 81]	40 [13, 73]	31 [8, 64]	23 [4.2, 53]
Optimistic	Weibull	5	32 [8.3, 64]	22 [3.5, 52]	14 [1.3, 39]	6.9 [0.27, 24]
Optimistic	Weibull	10	37 [10, 73]	26 [4.5, 60]	16 [1.6, 45]	8.4 [0.35, 29]
Optimistic	Weibull	20	40 [11, 75]	28 [4.9, 63]	17 [1.8, 48]	9.1 [0.37, 31]
Pessimistic	Lognormal	5	55 [20, 90]	41 [9.8, 81]	28 [4.1, 68]	17 [1.2, 51]
Pessimistic	Lognormal	10	53 [18, 88]	39 [9.2, 79]	27 [3.8, 66]	16 [1.1, 48]
Pessimistic	Lognormal	20	51 [18, 86]	38 [8.6, 77]	26 [3.7, 64]	15 [1.1, 46]
Pessimistic	Pareto	5	64 [30, 93]	54 [20, 88]	44 [13, 81]	33 [6.6, 71]
Pessimistic	Pareto	10	62 [28, 91]	52 [19, 86]	42 [12, 79]	31 [6.3, 68]
Pessimistic	Pareto	20	60 [26, 90]	50 [18, 85]	40 [11, 77]	30 [5.9, 66]
Pessimistic	Weibull	5	54 [17, 89]	39 [7.9, 80]	25 [2.8, 66]	14 [0.6, 46]
Pessimistic	Weibull	10	51 [16, 88]	37 [7.2, 77]	24 [2.6, 63]	13 [0.55, 43]
Pessimistic	Weibull	20	50 [15, 86]	35 [6.9, 75]	23 [2.4, 61]	12 [0.52, 41]
Status Quo	Lognormal	5	46 [15, 81]	33 [7.3, 71]	22 [3, 57]	13 [0.9, 40]
Status Quo	Lognormal	10	46 [15, 81]	33 [7.2, 70]	22 [3, 57]	13 [0.86, 40]
Status Quo	Lognormal	20	45 [15, 80]	33 [7, 70]	22 [3, 57]	13 [0.86, 40]
Status Quo	Pareto	5	54 [22, 86]	45 [15, 79]	36 [9.2, 70]	26 [5, 59]
Status Quo	Pareto	10	54 [23, 85]	45 [15, 78]	35 [9.4, 70]	26 [4.9, 59]
Status Quo	Pareto	20	54 [22, 85]	44 [15, 79]	35 [9.1, 70]	26 [4.9, 58]
Status Quo	Weibull	5	44 [13, 81]	31 [5.8, 69]	20 [2.1, 54]	11 [0.43, 36]
Status Quo	Weibull	10	44 [13, 80]	31 [5.7, 68]	20 [2, 54]	10 [0.43, 36]
Status Quo	Weibull	20	44 [13, 80]	31 [5.6, 68]	20 [2, 54]	10 [0.43, 36]

Projections for the cumulative probability of at least one event occurring between 2019 and 2039 with severity for the Killed Gunfire variable meeting each of several thresholds in percentages. Each column presents results for a different severity threshold. The table provides a median prediction for each combination of distribution model, time window, and rate scenario. The bracketed numbers represent the 90% posterior

a different severity threshold and the rows cover different combinations of distribution model, time window, and rate scenario. For the severity of the Las Vegas shooting (60 gunfire fatalities), the results range from 25% [5.3–55] under the optimistic scenario for the lognormal model with a 5 year time window to about 55% [21–89] for the pessimistic scenario using the Pareto model with the 5 year time window. A mid-range prediction comes from the lognormal model under the status quo scenario, which projects a 35% [8.5, 72] probability. The probability of an attack with 100 gunfire fatalities occurring ranges from 7.0% [0.27, 24] for Weibull model in the optimistic scenario with the 5-year time window to 34% [7.2, 72] for the pessimistic Pareto with the 5 year window. Not surprisingly, the shortest time window (which included the Las Vegas event) yields the most extreme probabilities and, as we have seen, the Pareto model produces more pessimistic projections, particularly in the high severity domain.

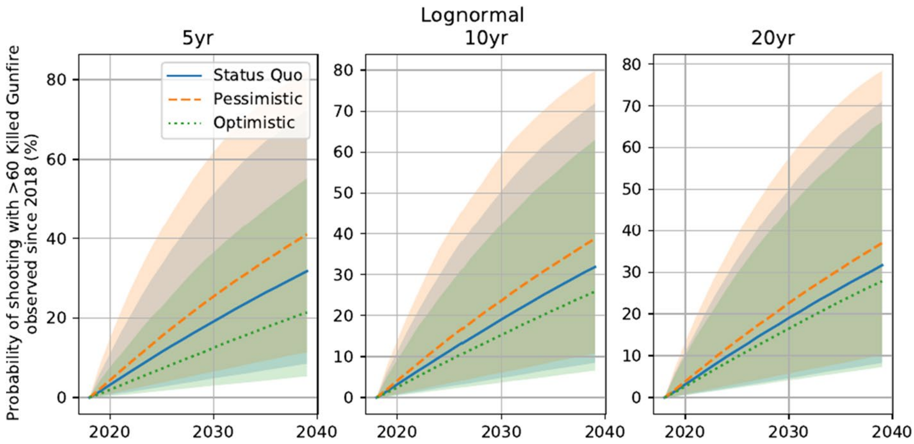

Fig. 4 Projections for events exceeding 60 Killed Gunfire fatalities, Lognormal model, tail location of 4. Projections for the cumulative probability of an incident occurring which exceeds the threshold of 60 Killed Gunfire fatalities between 2019 and 2039. The projections shown are for the lognormal tail model fit with tail location of 10 in each of the three rate scenarios (colored lines) for each of the three studied time windows (left, center, and right) described in the text

For the Total Shot variable, the probability of significantly exceeding the Las Vegas shooting's severity (using 500 victims as a test threshold) ranges from 0.8% [0.02, 3] for the optimistic Weibull model with the 5 year time window to 24% [6.4, 50] for the pessimistic Pareto at the 5 years window. Generally, the projected probabilities to repeat the fatality level of the Las Vegas shooting are higher than the probability of repeating its total shooting injuries, owing to the extraordinarily high level of Total Shot in that incident. Further statistics are reported in Table D3, and additional single-year and cumulative probabilities are reported elsewhere in "Appendix D".

Figure 4 illustrates the projection of the fitted lognormal distribution model with tail location of 10 for the Killed Gunfire variable over time with the severity threshold of 60 victims, again corresponding to the highest ever historical severity from the Las Vegas shooting. Each panel represents a choice of time window and each line within each panel represents a choice of rate scenario. Each line shows the cumulative probability, under the model, for any event meeting the threshold over time. The lines rise monotonically because they are cumulative probabilities. The shaded regions represent the 90% interval associated with each scenario. The uncertainty grows quickly with time as the posterior interval associated with the distribution model is compounded year over year. The shaded regions strongly overlap between the different scenarios because the uncertainties are generally larger than the differences between the rate estimate scenarios. We note that the position of the solid lines in Fig. 4, which are based on simulation medians, differ very slightly from the mean estimates reported in Table 3.

Finally, Fig. 5 compares the fitted probability distributions for each model for the Killed Gunfire variable at the two alternative values for the tail location. This comparison supports our choice to focus on the models with tail location of 10. In particular, the predicted distribution of gunfire fatalities for the tail location of 4 substantially under-predicts the actual occurrence of extreme events like the Las Vegas and Orlando shootings, with the lognormal and Weibull fits falling below the observed complementary cumulative distribution by nearly two orders of magnitude at $x = 50$. In general, the projections to higher

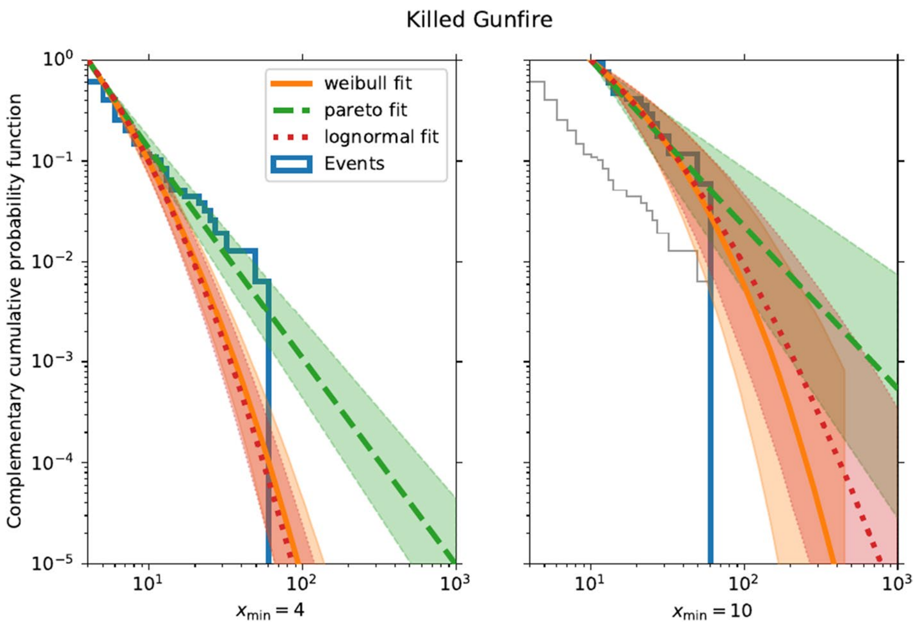

Fig. 5 Comparison between tail locations, Killed Gunfire. Comparison of modeled complementary cumulative distributions for the Killed Gunfire variable, as in Fig. 2, between models fit with tail locations of 4 (left) and 10 (right). As in Fig. 2, the light gray line (right facet) reflects the non-truncated, full data distribution

severity levels agree more across models at the tail location of 10 because, when low severity values are truncated from the distribution, there are fewer data points with leverage to influence the fit from the left side of the distribution. See "Appendix C" for further discussion of the impact of tail location.

Conclusions

Mass public shootings are a rare but fear-inducing phenomenon that, because of their seemingly random nature, have been thought to be beyond prediction and projection. Recent scholarship has illustrated how to fit heavy tailed distributions to rare events to forecast severity into the future. In this study, we sought to utilize those techniques to forecast the severity of mass public shootings up to the year 2039 using a variety of assumptions.

Our results illustrate that forward projections of incident rates for high severity mass public shooting events strongly depend on assumptions made about the tail shape of the severity distribution, and even modeling details such as the tail location. Compared to the case of terrorist events, as studied by Clauset and Woodard (2013), mass public shootings have historically been sufficiently rare that the distribution statistics are very poorly sampled at high severity rates. As a result, the historical data have relatively low constraining power for models of their long tail behavior.

In particular, we find that extrapolations to severities at least as high as the most extreme events previously observed vary significantly depending on the choice of distribution model. Pareto model fits tend to be substantially more pessimistic (predicting more frequent extremely severe events) than Weibull or lognormal fits (see Figs. 2 and 5). Empirical measures of goodness of fit do not solve this problem, as they have little power to distinguish between these model alternatives (Fig. 3).

While the power law Pareto distribution may be scale-free in principle, an attractive characteristic suggesting the possibility of free extrapolation to increasingly extreme values, this benefit is not realized in practice. The issue manifests such that the choice of scale location substantially influences the probabilities predicted by the Pareto model. When lower severity data are admitted to the modeling procedure, the predictions of the Pareto model become substantially more optimistic (Fig. 5). The Weibull and Lognormal, which are inherently two-parameter models in addition to the tail location variable, are even more sensitive to the choice of this data selection parameter.

Finally, although the projection uncertainties reported in Tables 3, 4, 5 and 6 and "Appendix B1–B3" are large, they are likely underestimates. The predictive uncertainty represented by these intervals as well as the shaded regions in Figs. 2 and 5 are posterior intervals conditioned on the model selection and do not directly incorporate variance between models. An alternate approach, which may be useful to establish an upper bound on the model-selection uncertainty, would be to fit a non-parametric model such as a Gaussian Process with no constraints (or very few constraints, such as monotonicity). Additionally, while our approach accounts for growth in the U.S. population, we assume the underlying per capita rate of mass public shootings is constant over time. Any long-term change in the incident rate would alter the probability estimates we report here and uncertainty in the growth trend would enlarge our prediction intervals.

While forecasting the probability of severe events cannot tell us where or exactly when those events may transpire, such analyses are important for policy and practice. There is much confusion around what constitutes mass shootings and why they occur. As a result, there is pessimism regarding whether mass public shootings can be predicted (Archer 2018). However, most efforts at prediction rely on developing risk assessments or profiles of potential shooters. By using sound methodology to predict the probability of mass casualty events occurring in the future, our study provides estimates that can be used to inform a variety of resource allocation decisions. Such estimates may inform, for example, public health officials modeling the trauma capacity of regional hospital systems, policymakers seeking to understand the potential consequences of the availability of high capacity magazines and assault weapons, or public safety officers assessing risk around large public gatherings.

Our findings suggest the probability that a mass public shooting at the largest previously-observed scale will reoccur in the U.S. is not trivial. Indeed, despite the variability across the forecasts presented here, the results still suggest that the likelihood the U.S. will endure another attack as deadly as the 2017 Las Vegas massacre at some point over the next 20 years is greater than five percent, even under the most optimistic modeling scenario and assuming the lower uncertainty bound (5% posterior interval) of our model (Table 3). Therefore, given the devastating impact that catastrophic mass public shootings have on society, policymakers will need to ensure there are sufficient resources and policies in place to help reduce the incidence and severity of lethal mass violence.

Appendix A

See Table 4.

Table 4 Mass public shootings, 1976–2018

Case	Date	City	State	Killed	Killed gunfire	Injured	Total shot
1	7/12/1976	Fullerton	CA	7	7	2	9
2	2/14/1977	New Rochelle	NY	6	6	4	10
3	7/23/1977	Klamath Falls	OR	6	6	2	8
4	8/26/1977	Hackettstown	NJ	6	6	0	6
5	6/17/1978	Warwick	RI	4	4	0	4
6	2/3/1980	El Paso	TX	5	5	3	8
7	6/22/1980	Daingerfield	TX	5	5	10	15
8	7/21/1980	Coraopolis	PA	4	4	1	5
9	8/1/1980	Holmes Beach	FL	4	4	1	5
10	5/7/1981	Salem	OR	4	4	19	23
11	10/16/1981	Floyd State Police	KY	5	5	3	8
12	1/2/1982	Elyria	OH	4	4	0	4
13	5/3/1982	Anchorage	AK	4	4	0	4
14	8/9/1982	Grand Prairie	TX	6	6	4	10
15	8/20/1982	Miami	FL	8	8	3	11
16	9/6/1982	Noyes Island	AK	8	8	0	8
17	3/1/1983	State Police	AK	6	6	2	8
18	10/11/1983	Waller	TX	6	6	0	6
19	5/17/1984	State Police	AK	7	7	0	7
20	6/29/1984	Dallas	TX	6	6	1	7
21	7/18/1984	San Ysidro	CA	21	21	19	40
22	7/24/1984	Hot Springs	AR	4	4	3	7
23	9/9/1984	Oakley	ID	4	4	1	5
24	3/16/1985	South Connellsville	PA	4	4	1	5
25	8/20/1986	Edmond	OK	14	14	6	20
26	4/23/1987	Palm Bay	FL	6	6	10	16
27	2/16/1988	Sunnyvale	CA	7	7	5	12
28	7/17/1988	Forsythe	NC	4	4	5	9
29	9/22/1988	Chicago	IL	4	4	2	6
30	12/7/1988	San Luis Obispo	CA	42	4	0	4
31	1/17/1989	Stockton	CA	5	5	31	36
32	9/14/1989	Louisville	KY	8	8	12	20
33	6/18/1990	Jacksonville	FL	9	9	4	13
34	10/16/1991	Killeen	TX	23	23	22	45
35	11/1/1991	Iowa City	IA	5	5	1	6
36	11/9/1991	Harrodsburg	KY	4	4	2	6
37	11/14/1991	Royal Oak	MI	4	4	8	12
38	12/9/1991	Moniteau	MO	4	4	1	5
39	5/1/1992	Olivehurst	CA	4	4	10	14
40	10/15/1992	Schuyler	NY	4	4	0	4
41	11/8/1992	Morro Bay	CA	6	6	1	7
42	7/1/1993	San Francisco	CA	8	8	6	14
43	7/8/1993	Jackson/Greenville	MS	5	5	0	5
44	8/6/1993	Fayetteville	NC	4	4	8	12

Table 4 (continued)

Case	Date	City	State	Killed	Killed gunfire	Injured	Total shot
45	10/14/1993	El Cajon	CA	4	4	2	6
46	12/2/1993	Oxnard	CA	4	4	4	8
47	12/7/1993	Long Island Rl	NY	6	6	17	23
48	12/14/1993	Aurora	CO	4	4	1	5
49	6/20/1994	Spokane	WA	4	4	23	27
50	4/3/1995	Corpus Christi	TX	5	5	1	6
51	7/19/1995	Los Angeles	CA	4	4	0	4
52	12/19/1995	New York	NY	5	5	3	8
53	2/9/1996	Fort Lauderdale	FL	5	5	1	6
54	4/25/1996	Jackson	MS	5	5	3	8
55	8/20/1997	Colebrook	NH	4	4	4	8
56	9/15/1997	Aiken	SC	4	4	3	7
57	12/18/1997	Orange	CA	4	4	2	6
58	3/6/1998	Newington	CT	4	4	0	4
59	3/24/1998	Craighead	AR	5	5	10	15
60	5/21/1998	Springfield	OR	4	4	25	29
61	3/10/1999	Gonzalez	LA	4	4	4	8
62	4/20/1999	Jefferson	CO	13	13	25	38
63	6/3/1999	Las Vegas	NV	4	4	1	5
64	7/29/1999	Atlanta	GA	12	9	13	22
65	9/15/1999	Fort Worth	TX	7	7	7	14
66	11/2/1999	Honolulu	HI	7	7	0	7
67	12/30/1999	Tampa	FL	5	5	3	8
68	3/20/2000	Irving	TX	5	5	0	5
69	4/28/2000	Pittsburgh	PA	5	5	1	6
70	12/26/2000	Wakefield	MA	7	7	0	7
71	1/9/2001	Houston	TX	4	4	0	4
72	2/5/2001	Melrose	IL	4	4	4	8
73	7/3/2001	Rifle	CO	4	4	3	7
74	9/8/2001	Sacramento	CA	5	5	2	7
75	3/22/2002	South Bend	IN	4	4	2	6
76	2/25/2003	Huntsville	AL	4	4	1	5
77	7/8/2003	Lauderdale	MS	6	6	8	14
78	8/27/2003	Chicago	IL	6	6	0	6
79	10/24/2003	Bonner City	ID	4	4	0	4
80	7/2/2004	Kansas City	KS	5	5	2	7
81	11/21/2004	Meteor	WI	6	6	2	8
82	12/8/2004	Columbus	OH	4	4	7	11
83	3/11/2005	Atlanta	GA	4	4	2	6
84	3/12/2005	Brookfield	WI	7	7	4	11
85	3/21/2005	Red Lake	MN	9	9	5	14
86	8/28/2005	Sash	TX	4	4	0	4
87	1/30/2006	Goleta	CA	7	7	0	7
88	3/25/2006	Seattle	WA	6	6	2	8

Table 4 (continued)

Case	Date	City	State	Killed	Killed gunfire	Injured	Total shot
89	4/18/2006	St. Louis	MO	4	4	1	5
90	5/21/2006	Baton Rouge	LA	5	5	1	6
91	10/2/2006	Nickel Mines	PA	5	5	5	10
92	2/12/2007	Salt Lake City	UT	5	5	4	9
93	4/16/2007	Blacksburg	VA	32	32	17	49
94	7/22/2007	Philadelphia	PA	4	4	0	4
95	11/22/2007	Unity	MD	4	4	0	4
96	12/5/2007	Omaha	NE	8	8	4	12
97	12/9/2007	Colorado Springs	CO	4	4	5	9
98	2/7/2008	Kirkwood	MO	6	6	1	7
99	2/14/2008	Dekalb	IL	5	5	21	26
100	3/18/2008	Santa Maria	CA	4	4	0	4
101	6/25/2008	Henderson	KY	5	5	1	6
102	9/2/2008	Alger	WA	6	6	4	10
103	2/14/2009	Brockport	NY	4	4	0	4
104	3/29/2009	Carthage	NC	8	8	2	10
105	4/3/2009	Binghamton	NY	13	13	4	17
106	11/1/2009	Mount Airy	NC	4	4	0	4
107	11/5/2009	Killeen	TX	13	13	32	45
108	11/29/2009	Parkland	WA	4	4	0	4
109	1/12/2010	Kennesaw	GA	4	4	1	5
110	4/3/2010	Los Angeles	CA	4	4	2	6
111	6/6/2010	Hialeah	FL	4	4	3	7
112	8/3/2010	Manchester	CT	8	8	2	10
113	8/14/2010	Buffalo	NY	4	4	4	8
114	1/8/2011	Tucson	AZ	6	6	14	20
115	7/23/2011	Grand Prairie	TX	5	5	4	9
116	9/6/2011	Carson City	NV	4	4	7	11
117	10/12/2011	Seal Beach	CA	8	8	1	9
118	2/21/2012	Norcross	GA	4	4	0	4
119	4/2/2012	Oakland	CA	7	7	3	10
120	5/30/2012	Seattle	WA	5	5	1	6
121	7/20/2012	Aurora	CO	12	12	70	82
122	8/5/2012	Oak Creek	WI	6	6	3	9
123	9/27/2012	Minneapolis	MN	6	6	2	8
124	12/14/2012	Newtown	CT	27	27	2	29
125	3/13/2013	Herkimer	NY	4	4	2	6
126	6/7/2013	Santa Monica	CA	5	5	1	6
127	7/26/2013	Hialeah	FL	6	6	0	6
128	9/16/2013	Washington	DC	12	12	3	15
129	2/20/2014	Alturas	CA	4	4	2	6
130	5/23/2014	Isla Vista	CA	6	6	14	20
131	10/24/2014	Marysville	WA	4	4	3	7
132	6/17/2015	Charleston	SC	9	9	1	10

Table 4 (continued)

Case	Date	City	State	Killed	Killed gunfire	Injured	Total shot
133	7/16/2015	Chattanooga	TN	5	5	2	7
134	10/1/2015	Roseburg	OR	9	9	9	18
135	12/2/2015	San Bernardino	CA	14	14	24	38
136	2/20/2016	Kalamazoo	MI	6	6	2	8
137	6/12/2016	Orlando	FL	49	49	53	102
138	7/7/2016	Dallas	TX	5	5	11	16
139	9/23/2016	Burlington	WA	5	5	0	5
140	1/6/2017	Fort Lauderdale	FL	5	5	6	11
141	2/6/2017	Yazoo City	MS	4	4	0	4
142	3/22/2017	Rothschild	WI	4	4	0	4
143	6/5/2017	Orlando	FL	5	5	0	5
144	10/1/2017	Las Vegas	NV	60	60	411	471
145	11/5/2017	Sutherland Springs	TX	25	25	20	45
146	11/14/2017	Rancho Tehama	CA	5	5	18	23
147	1/28/2018	Melcroft	PA	4	4	1	5
148	2/14/2018	Parkland	FL	17	17	17	34
149	2/26/2018	Detroit	MI	4	4	0	4
150	4/22/2018	Nashville	TN	4	4	4	8
151	5/18/2018	Santa Fe	TX	10	10	13	23
152	5/30/2018	Scottsdale	AZ	6	6	0	6
153	6/28/2018	Annapolis	MD	5	5	2	7
154	9/12/2018	Bakersfield	CA	5	5	0	5
155	10/27/2018	Pittsburgh	PA	11	11	6	17
156	11/7/2018	Thousand Oaks	CA	12	12	12	24

Appendix B: Implementation of Bayesian Tail Models in Stan

As described in the Methods section, we implemented Bayesian discrete Pareto and continuous truncated lognormal and Weibull models using the python interface (Stan Development Team 2018) to the probabilistic programming language Stan (Carpenter et al. 2017), version 2.19.1.1. We used the Hamiltonian Monte Carlo sampling method to draw samples from the model posteriors. Full code and data associated with this analysis will be posted on GitHub at https://github.com/(authors)/ and we briefly describe these methods here.

To implement the discrete Pareto (also called Zipf) model with arbitrary values of the tail location x_{min}, we implement an approximation of the Hurwitz zeta function in C++ and imported it as an external library to the Stan model. We apply a normal prior on the Pareto rate parameter α with mean 2 and standard deviation 2. While the normal is a relatively strong prior distribution in the sense of having weak tails that do not support extreme parameter samples, we note that none of our model fits has a significant number of samples above the 1 sigma level of this prior distribution (i.e. α is generally much less than 4).

To implement the continuous truncated lognormal, we use the Stan truncated distribution notation to declare a lower bound on the lognormal distribution. We apply half-Cauchy prior distributions to the location (μ) and scale parameters (σ), where the half-Cauchy scale is set to the standard value (1).

To implement the continuous truncated Weibull, we again use the Stan truncated distribution notation and apply standard half-Cauchy prior distributions to the shape (α) and scale parameters (σ).

To implement model comparison via Pareto-smoothed importance sampling leave-one-out cross validation (PSIS-LOO; Vehtari et al. 2017), we add a pointwise log likelihood calculation to the generated quantities block of each model and then supply these values to the *loo* method of the Arviz python package (Kumar et al. 2019) with the scale parameter set to “log”. To calculate the ELPD over data points at $X > 10$ only, generate an additional pointwise log likelihood variable for this subset of the data.

To facilitate plotting, we implement identical probability models for the discrete Pareto, truncated Weibull, and truncated lognormal in python using the scipy library (Virtanen et al. 2020).

What follows is the Stan code for each model. The C++ implementation of the Hurwitz zeta function is available in the GitHub repository.

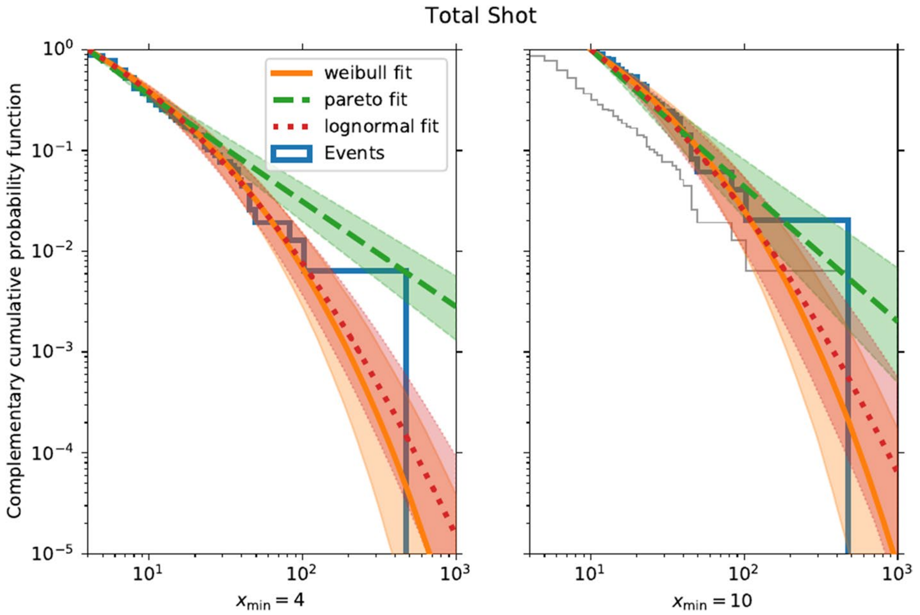

Fig. 6 Comparison between tail locations, Total Shot. Comparison of fitted severity probability distributions with different tail locations, as in Fig. 5, for the Total Shot variable

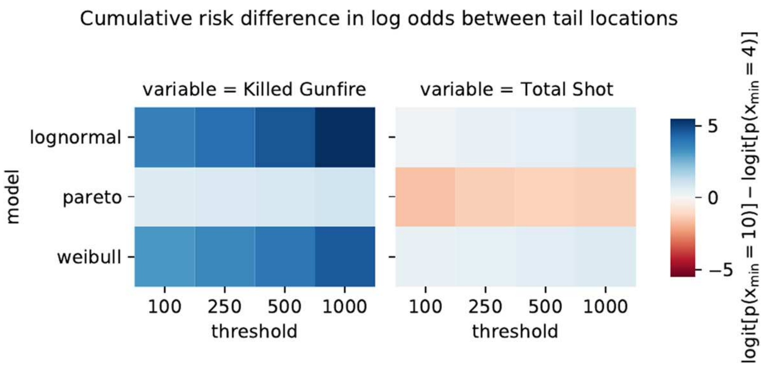

Fig. 7 Forecast comparisons across tail locations. Comparison of forecasted log odds for the cumulative risk (probability) of extreme events between 2019 and 2039 for each variable (left and right facets) with various thresholds (x-axis values) across models (y-axis). The color scale shows the difference in log odds between the models with tail locations of 10 and 4

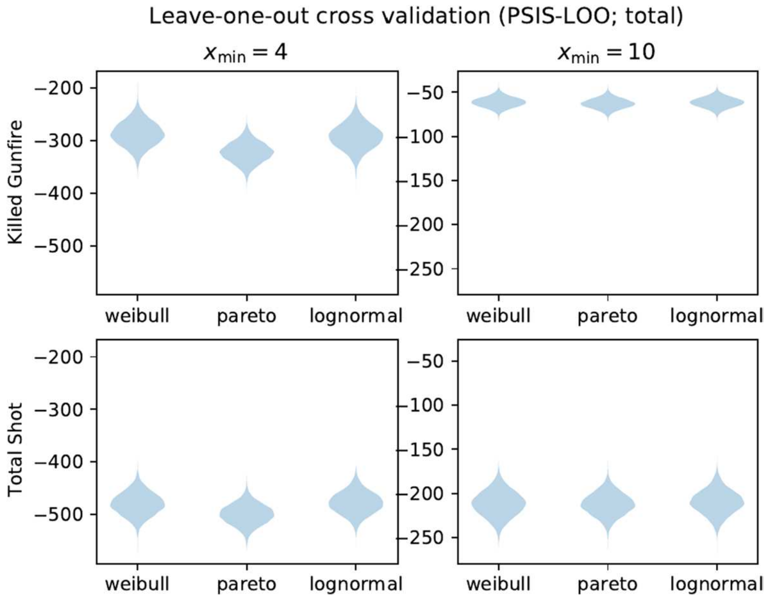

Fig. 8 Model evaluation for all datapoints. Model performance estimates, as in Fig. 3, but using all datapoints to calculate the ELPD

Algorithm B1 Stan code for Lognormal tail model

```
data {
    int<lower=0> N; // Number of events
    vector[N] y; // Number of days since event
    int<lower=0> y_min; // minimum y-value
}
parameters {
    real<lower=0> sigma;
    real<lower=0> mu;
}
model {
    for (i in 1:N) {
        y[i] ~ lognormal(mu, sigma) T[y_min, ];
    }
    mu ~ cauchy(0,1);
    sigma ~ cauchy(0,1);
}
generated quantities {
    vector[N] log_likelihood;
    for (n in 1:N) {
        if (y[n] < y_min) {
            log_likelihood[n] = negative_infinity();
        } else {
            log_likelihood[n] = lognormal_lpdf(y[n] | mu, sigma) -
lognormal_lccdf(y_min| mu, sigma);
        }
    }
}
```

Table 5 Event probability projections: killed gunfire, single year, tail location of 4

Scenario	Model	Window	$P_{2019}\ (x>49)$	$P_{2019}\ (x>60)$	$P_{2019}\ (x>75)$	$P_{2019}\ (x>100)$
Optimistic	Lognormal	5	0.071 [0, 0.21]	0.028 [0, 0.087]	0.0094 [0, 0.032]	0.0022 [0, 0.0078]
Optimistic	Lognormal	10	0.088 [0.0084, 0.26]	0.034 [0.0027, 0.11]	0.012 [0.0007, 0.038]	0.0027 [0.0001, 0.0095]
Optimistic	Lognormal	20	0.096 [0.01, 0.28]	0.037 [0.0033, 0.11]	0.013 [0.00087, 0.041]	0.003 [0.00013, 0.01]
Optimistic	Pareto	5	1.4 [0, 3.5]	0.96 [0, 2.4]	0.62 [0, 1.6]	0.36 [0, 0.95]
Optimistic	Pareto	10	1.8 [0.32, 4.2]	1.2 [0.2, 2.9]	0.77 [0.13, 1.9]	0.44 [0.066, 1.1]
Optimistic	Pareto	20	1.9 [0.38, 4.5]	1.3 [0.24, 3.1]	0.84 [0.15, 2]	0.48 [0.078, 1.2]
Optimistic	Weibull	5	0.1 [0, 0.31]	0.042 [0, 0.14]	0.015 [0, 0.053]	0.0039 [0, 0.015]
Optimistic	Weibull	10	0.13 [0.012, 0.38]	0.053 [0.0035, 0.17]	0.019 [0.00083, 0.065]	0.0048 [0.00011, 0.018]
Optimistic	Weibull	20	0.14 [0.014, 0.4]	0.057 [0.0045, 0.18]	0.021 [0.0011, 0.07]	0.0053 [0.00014, 0.019]
Pessimistic	Lognormal	5	0.15 [0.024, 0.42]	0.06 [0.0075, 0.18]	0.021 [0.002, 0.064]	0.0048 [0.00032, 0.016]
Pessimistic	Lognormal	10	0.14 [0.021, 0.4]	0.057 [0.0068, 0.17]	0.019 [0.0018, 0.06]	0.0045 [0.00029, 0.015]
Pessimistic	Lognormal	20	0.14 [0.019, 0.38]	0.053 [0.0062, 0.16]	0.018 [0.0016, 0.057]	0.0042 [0.00027, 0.014]
Pessimistic	Pareto	5	3.1 [0.86, 6.6]	2.1 [0.55, 4.6]	1.3 [0.34, 3.1]	0.77 [0.18, 1.8]
Pessimistic	Pareto	10	2.9 [0.78, 6.2]	1.9 [0.49, 4.3]	1.3 [0.3, 2.9]	0.72 [0.16, 1.7]
Pessimistic	Pareto	20	2.7 [0.7, 5.9]	1.8 [0.45, 4.1]	1.2 [0.27, 2.8]	0.68 [0.14, 1.6]
Pessimistic	Weibull	5	0.22 [0.033, 0.62]	0.092 [0.01, 0.28]	0.033 [0.0026, 0.11]	0.0085 [0.00035, 0.031]
Pessimistic	Weibull	10	0.21 [0.03, 0.58]	0.086 [0.0094, 0.26]	0.031 [0.0023, 0.1]	0.0079 [0.00032, 0.029]
Pessimistic	Weibull	20	0.2 [0.027, 0.55]	0.081 [0.0085, 0.25]	0.029 [0.0021, 0.096]	0.0075 [0.00029, 0.027]
Status Quo	Lognormal	5	0.11 [0.015, 0.32]	0.044 [0.0047, 0.13]	0.015 [0.0012, 0.048]	0.0035 [0.0002, 0.012]
Status Quo	Lognormal	10	0.11 [0.014, 0.32]	0.044 [0.0046, 0.13]	0.015 [0.0012, 0.048]	0.0035 [0.00019, 0.012]
Status Quo	Lognormal	20	0.11 [0.014, 0.31]	0.044 [0.0044, 0.13]	0.015 [0.0012, 0.047]	0.0034 [0.00019, 0.012]
Status Quo	Pareto	5	2.3 [0.51, 5.1]	1.5 [0.33, 3.5]	0.99 [0.2, 2.4]	0.56 [0.11, 1.4]
Status Quo	Pareto	10	2.2 [0.5, 5]	1.5 [0.32, 3.5]	0.98 [0.2, 2.3]	0.56 [0.11, 1.4]
Status Quo	Pareto	20	2.2 [0.5, 5]	1.5 [0.32, 3.5]	0.98 [0.19, 2.3]	0.56 [0.1, 1.4]
Status Quo	Weibull	5	0.16 [0.02, 0.47]	0.068 [0.0063, 0.21]	0.025 [0.0015, 0.082]	0.0062 [0.00021, 0.023]
Status Quo	Weibull	10	0.16 [0.02, 0.46]	0.067 [0.0062, 0.21]	0.024 [0.0015, 0.08]	0.0062 [0.00021, 0.023]
Status Quo	Weibull	20	0.16 [0.019, 0.46]	0.067 [0.0061, 0.21]	0.024 [0.0015, 0.081]	0.0061 [0.0002, 0.022]

Projections for the single-year probability of at least one event occurring in 2019 with severity for the Killed Gunfire variable meeting each of several thresholds, for models with a tail location of $x_{min} = 4$. Formatting follows Table 3

Algorithm B2 Stan code for Pareto tail model

```
functions{
    real hurwitz_zeta(real s, real a);
}
data{
  int<lower=0> N; // number of datapoints
  int<lower=0> y_min; // minimum y-value
  int<lower=y_min> y[N]; // Datapoints
}
transformed data{
  int<lower=0> K = max(y) - min(y) + 1; // number of unique values
  int values[K]; // y-values
  int<lower=0> frequencies[K]; // number of counts at each y-value
  real real_xmin; // needed for C+ Hurwitz zeta

  for (k in 1:K) {
      values[k] = min(y) + k - 1;
      frequencies[k] = 0;
  }
  for (n in 1:N) {
      int k;
      k = y[n] - min(y) + 1;
      frequencies[k] += 1;
  }
  real_xmin = round(x_min);
}
parameters{
  real <lower=1> alpha;
}
model{
  real constant = log(hurwitz_zeta(alpha, real_xmin));
  for (k in 1:K) {
    target += frequencies[k] * (-alpha * log(values[k]) - constant);
  }
  alpha ~ normal(2, 2);
}
/*TODO: implement rng for prior and posterior predictive checks*/
generated quantities {
    vector[N] log_likelihood;
    for (n in 1:N) {
        log_likelihood[n] = -alpha*log(y[n]) - log(hurwitz_zeta(alpha,
real_xmin));
    }
}
```

Table 6 Event probability projections: total shot, single year, tail location of 4

Scenario	Model	Window	P ($x>100$)	P_{2019} ($x>250$)	P_{2019} (>500)	P_{2019} ($x>1000$)
Optimistic	Lognormal	5	2.3 [0, 5.7]	0.29 [0, 0.83]	0.049 [0, 0.16]	0.0072 [0, 0.026]
Optimistic	Lognormal	10	2.9 [0.51, 6.8]	0.36 [0.041, 1]	0.061 [0.0038, 0.19]	0.009 [0.00027, 0.032]
Optimistic	Lognormal	20	3.1 [0.61, 7.2]	0.39 [0.05, 1.1]	0.067 [0.005, 0.21]	0.0098 [0.00035, 0.035]
Optimistic	Pareto	5	8.7 [0, 19]	3.5 [0, 8.2]	1.8 [0, 4.3]	0.89 [0, 2.3]
Optimistic	Pareto	10	11 [2.5, 22]	4.4 [0.9, 9.7]	2.2 [0.41, 5.2]	1.1 [0.19, 2.7]
Optimistic	Pareto	20	12 [2.8, 23]	4.8 [1, 10]	2.4 [0.48, 5.5]	1.2 [0.22, 2.9]
Optimistic	Weibull	5	2.1 [0, 5.4]	0.19 [0, 0.62]	0.022 [0, 0.086]	0.002 [0, 0.0086]
Optimistic	Weibull	10	2.6 [0.44, 6.4]	0.24 [0.014, 0.76]	0.027 [0.0003, 0.11]	0.0024 [0, 0.011]
Optimistic	Weibull	20	2.9 [0.52, 6.9]	0.25 [0.018, 0.81]	0.029 [0.00042, 0.11]	0.0027 [0, 0.012]
Pessimistic	Lognormal	5	5 [1.4, 11]	0.63 [0.11, 1.6]	0.11 [0.011, 0.33]	0.016 [0.00085, 0.054]
Pessimistic	Lognormal	10	4.6 [1.3, 10]	0.59 [0.1, 1.6]	0.1 [0.01, 0.3]	0.015 [0.00077, 0.051]
Pessimistic	Lognormal	20	4.4 [1.1, 9.6]	0.55 [0.091, 1.5]	0.094 [0.0094, 0.29]	0.014 [0.00071, 0.048]
Pessimistic	Pareto	5	18 [6.6, 33]	7.6 [2.4, 15]	3.8 [1.1, 8.1]	1.9 [0.51, 4.3]
Pessimistic	Pareto	10	17 [5.9, 31]	7.1 [2.2, 14]	3.6 [1, 7.7]	1.8 [0.46, 4.1]
Pessimistic	Pareto	20	16 [5.3, 30]	6.7 [2, 14]	3.4 [0.9, 7.2]	1.7 [0.42, 3.8]
Pessimistic	Weibull	5	4.6 [1.2, 10]	0.41 [0.041, 1.3]	0.047 [0.0011, 0.18]	0.0042 [0, 0.019]
Pessimistic	Weibull	10	4.3 [1, 9.6]	0.38 [0.037, 1.2]	0.044 [0.00098, 0.17]	0.004 [0, 0.017]
Pessimistic	Weibull	20	4 [0.95, 9.1]	0.36 [0.034, 1.1]	0.041 [0.0009, 0.16]	0.0037 [0, 0.016]
Status Quo	Lognormal	5	3.7 [0.82, 8.3]	0.46 [0.068, 1.3]	0.079 [0.0071, 0.24]	0.012 [0.00052, 0.04]
Status Quo	Lognormal	10	3.6 [0.81, 8.2]	0.46 [0.067, 1.2]	0.078 [0.0069, 0.24]	0.011 [0.00051, 0.04]
Status Quo	Lognormal	20	3.6 [0.79, 8.2]	0.46 [0.066, 1.2]	0.078 [0.0069, 0.24]	0.011 [0.00049, 0.04]
Status Quo	Pareto	5	14 [3.7, 26]	5.6 [1.4, 12]	2.8 [0.66, 6.3]	1.4 [0.31, 3.3]
Status Quo	Pareto	10	13 [3.7, 26]	5.5 [1.4, 12]	2.8 [0.65, 6.2]	1.4 [0.3, 3.3]
Status Quo	Pareto	20	13 [3.6, 26]	5.5 [1.4, 12]	2.8 [0.64, 6.2]	1.4 [0.3, 3.2]
Status Quo	Weibull	5	3.4 [0.69, 7.9]	0.3 [0.025, 0.94]	0.035 [0.00064, 0.13]	0.0031 [0, 0.014]
Status Quo	Weibull	10	3.3 [0.69, 7.8]	0.3 [0.025, 0.94]	0.034 [0.00063, 0.13]	0.0031 [0, 0.013]
Status Quo	Weibull	20	3.3 [0.68, 7.8]	0.3 [0.024, 0.92]	0.034 [0.00062, 0.13]	0.0031 [0, 0.013]

Projections for the single-year probability of at least one event occurring in 2019 with severity for the Total Shot variable meeting each of several threshold, for models with a tail location of $x_{min}=4$. Formatting follows Table 3

Algorithm B3 Stan code for Weibull tail model

```
data {
    int<lower=0> N; // Number of events
    vector[N] y; // Number of fatalities per event
    int<lower=0> y_min; // minimum y-value
}
parameters {
    real<lower=0> alpha;
    real<lower=0> sigma;
}
model {
    for (n in 1:N) {
        y[n] ~ weibull(alpha, sigma) T[y_min, ];
    }
    alpha ~ normal(0, 1);
    sigma ~ normal(0, 1);
}
generated quantities {
    vector[N] log_likelihood;
    for (n in 1:N) {
        if (y[n] < y_min) {
            log_likelihood[n] = negative_infinity();
        } else {
            log_likelihood[n] = weibull_lpdf(y[n] | alpha, sigma) -
weibull_lccdf(y_min| alpha, sigma);
        }
    }
}
```

Appendix C: Results for Different Tail Locations

See Tables 5, 6, 7 and 8 and Figs. 6, 7 and 8.

As discussed in the Methods section, we fit models with tail locations (distribution minimum values) of 4 and 10. We focused on the tail location of 10 for the results presented in the body of this article and we present further information about the tail location of 4 and the comparison between these models in this Appendix.

Moving the tail location from 4 to 10 generally increases the probability distributed to high severity values for both the Killed Gunfire and Total Shot variables. The comparison between the fitted probability models is shown in Fig. 5 (Killed Gunfire) and Fig. 6 (Total Shot). Figure 7 summarizes these comparisons by showing the difference in the forecasted cumulative probability between the models with tail location of 10 and 4. As described in the text, the comparison illustrates that the higher tail location substantially increases the forecast probability for the lognormal and Weibull models for Killed Gunfire, especially at higher severity thresholds. The effect of tail location has a much lesser impact on the Pareto model, though it still increases the probability somewhat. The impact of the tail location is generally much smaller on the Total Shot forecasts. The higher tail location slightly increases the forecast probability for the lognormal and Pareto models and slightly decreases the forecast probability of the Pareto model.

Figure 8 repeats the model evaluation measurement shown in Fig. 3 for the full dataset. In this figure, the ELPD values for the $x_{min}=4$ and $x_{min}=10$ models should not be compared directly because the choice of tail location truncates the full data distribution.

Table 7 Event probability projections: killed gunfire, cumulative, tail location of 4

Scenario	Model	Window	$P_{2019–2039}$ $(x>49)$	$P_{2019–2039}$ $(x>60)$	$P_{2019–2039}$ $(x>75)$	$P_{2019–2039}$ $(x>100)$
Optimistic	Lognormal	5	1.5 [0.31, 3.6]	0.58 [0.096, 1.6]	0.2 [0.025, 0.57]	0.046 [0.004, 0.14]
Optimistic	Lognormal	10	1.8 [0.38, 4.5]	0.72 [0.12, 1.9]	0.25 [0.031, 0.71]	0.057 [0.005, 0.18]
Optimistic	Lognormal	20	2 [0.41, 4.9]	0.78 [0.13, 2.1]	0.27 [0.034, 0.78]	0.062 [0.0055, 0.2]
Optimistic	Pareto	5	25 [12, 43]	18 [7.7, 32]	12 [4.7, 23]	7.1 [2.4, 15]
Optimistic	Pareto	10	30 [15, 50]	22 [9.5, 38]	15 [5.8, 28]	8.7 [3, 18]
Optimistic	Pareto	20	33 [16, 53]	23 [10, 41]	16 [6.3, 30]	9.5 [3.3, 19]
Optimistic	Weibull	5	2.1 [0.43, 5.4]	0.89 [0.13, 2.5]	0.32 [0.032, 0.99]	0.081 [0.0043, 0.28]
Optimistic	Weibull	10	2.6 [0.53, 6.6]	1.1 [0.16, 3.1]	0.4 [0.04, 1.2]	0.1 [0.0055, 0.34]
Optimistic	Weibull	20	2.8 [0.58, 7.1]	1.2 [0.18, 3.3]	0.43 [0.043, 1.3]	0.11 [0.006, 0.38]
Pessimistic	Lognormal	5	3.2 [0.68, 7.7]	1.3 [0.21, 3.3]	0.43 [0.056, 1.2]	0.1 [0.0089, 0.32]
Pessimistic	Lognormal	10	3 [0.64, 7.1]	1.2 [0.2, 3.1]	0.4 [0.052, 1.2]	0.094 [0.008, 0.3]
Pessimistic	Lognormal	20	2.8 [0.6, 6.8]	1.1 [0.19, 2.9]	0.38 [0.048, 1.1]	0.088 [0.0078, 0.28]
Pessimistic	Pareto	5	46 [25, 70]	34 [16, 57]	24 [10, 43]	15 [5.4, 29]
Pessimistic	Pareto	10	44 [23, 68]	33 [15, 55]	23 [9.5, 41]	14 [5, 27]
Pessimistic	Pareto	20	42 [22, 66]	31 [14, 53]	22 [8.8, 39]	13 [4.7, 26]
Pessimistic	Weibull	5	4.5 [0.94, 11]	1.9 [0.29, 5.2]	0.7 [0.072, 2.1]	0.18 [0.0096, 0.61]
Pessimistic	Weibull	10	4.2 [0.88, 11]	1.8 [0.27, 4.9]	0.65 [0.066, 2]	0.17 [0.009, 0.57]
Pessimistic	Weibull	20	4 [0.83, 9.8]	1.7 [0.25, 4.7]	0.61 [0.062, 1.9]	0.16 [0.0083, 0.53]
Status Quo	Lognormal	5	2.3 [0.5, 5.7]	0.92 [0.16, 2.4]	0.32 [0.041, 0.91]	0.074 [0.0063, 0.23]
Status Quo	Lognormal	10	2.3 [0.49, 5.7]	0.92 [0.15, 2.4]	0.31 [0.04, 0.89]	0.073 [0.0063, 0.23]
Status Quo	Lognormal	20	2.3 [0.49, 5.6]	0.91 [0.15, 2.4]	0.31 [0.04, 0.89]	0.072 [0.0063, 0.23]
Status Quo	Pareto	5	37 [19, 59]	27 [12, 46]	18 [7.5, 34]	11 [3.9, 22]
Status Quo	Pareto	10	37 [19, 58]	27 [12, 46]	18 [7.3, 34]	11 [3.9, 22]
Status Quo	Pareto	20	37 [18, 58]	27 [12, 46]	18 [7.3, 34]	11 [3.8, 22]
Status Quo	Weibull	5	3.3 [0.68, 8.4]	1.4 [0.21, 3.9]	0.51 [0.052, 1.5]	0.13 [0.0069, 0.44]
Status Quo	Weibull	10	3.3 [0.68, 8.4]	1.4 [0.21, 3.8]	0.51 [0.052, 1.6]	0.13 [0.0068, 0.45]
Status Quo	Weibull	20	3.3 [0.67, 8.4]	1.4 [0.21, 3.8]	0.5 [0.05, 1.6]	0.13 [0.0069, 0.44]

Projections for the cumulative (2019–2039) probability of at least one event occurring with severity for the Killed Gunfire variable meeting each of several threshold, for models with a tail location of $x_{min}=4$. For-

Table 7 (continued)

matting follows Table 3

(In contrast, Fig. 3 shows the ELPD on the subset of data where $x > = 10$, eliminating this truncation.) Fig. 8 illustrates the limited ability of the predictive performance statistic to discriminate between the models, with the ELPD distributions overlapping between models for each choice of variable and tail location. The greatest separation occurs for the tail location of 4, where the Pareto model is assessed to have somewhat lower performance because of its slightly less precise fit to the left side of the data distribution. In this case, the estimated ELPD is lower for the Pareto model than the alternatives in 87% of simulations for the Killed Gunfire variable and in 77% of simulations for Total Shot. This is due to the small number of observations at high severity levels having limited impact on the model likelihood, and the other models performing slightly better in fitting the more numerous observations at lower severity ($x < 10$).

Tables 5 show the projection results for the models with tail location of 4, formatted identically to Table 3 from the body of the article.

Table 8 Event probability projections: total shot, cumulative, tail location of 4

Scenario	Model	Window	$P_{2019–2039}$ ($x>100$)	$P_{2019–2039}$ ($x>250$)	$P_{2019–2039}$ ($x>500$)	$P_{2019–2039}$ ($x>1000$)
Optimistic	Lognormal	5	38 [19, 60]	5.8 [1.5, 13]	1 [0.14, 2.8]	0.15 [0.011, 0.49]
Optimistic	Lognormal	10	44 [23, 68]	7.2 [1.8, 16]	1.3 [0.18, 3.5]	0.19 [0.013, 0.61]
Optimistic	Lognormal	20	47 [25, 71]	7.8 [2, 18]	1.4 [0.2, 3.8]	0.2 [0.014, 0.67]
Optimistic	Pareto	5	83 [67, 95]	52 [32, 73]	31 [15, 50]	17 [7.2, 31]
Optimistic	Pareto	10	89 [76, 98]	59 [38, 80]	36 [19, 58]	20 [9, 37]
Optimistic	Pareto	20	91 [78, 98]	62 [41, 83]	39 [21, 61]	22 [9.8, 39]
Optimistic	Weibull	5	35 [15, 59]	3.8 [0.51, 11]	0.45 [0.013, 1.7]	0.041 [9.8e-05, 0.18]
Optimistic	Weibull	10	41 [19, 67]	4.7 [0.63, 13]	0.56 [0.016, 2.1]	0.051 [0.00012, 0.22]
Optimistic	Weibull	20	43 [20, 70]	5.1 [0.69, 14]	0.61 [0.018, 2.2]	0.056 [0.00013, 0.24]
Pessimistic	Lognormal	5	62 [37, 86]	12 [3.2, 27]	2.2 [0.32, 6.1]	0.33 [0.023, 1.1]
Pessimistic	Lognormal	10	60 [35, 85]	11 [3.1, 25]	2.1 [0.3, 5.7]	0.31 [0.022, 1]
Pessimistic	Lognormal	20	58 [33, 83]	11 [2.9, 24]	1.9 [0.28, 5.5]	0.29 [0.02, 0.95]
Pessimistic	Pareto	5	97 [92, 100]	78 [58, 94]	54 [31, 77]	32 [15, 55]
Pessimistic	Pareto	10	97 [90, 100]	76 [55, 93]	52 [30, 75]	31 [14, 52]
Pessimistic	Pareto	20	96 [89, 100]	74 [53, 92]	49 [28, 73]	29 [14, 50]
Pessimistic	Weibull	5	59 [31, 86]	8 [1.1, 22]	0.97 [0.028, 3.6]	0.089 [0.00021, 0.38]
Pessimistic	Weibull	10	56 [29, 84]	7.5 [1, 21]	0.91 [0.027, 3.4]	0.083 [0.00021, 0.36]
Pessimistic	Weibull	20	54 [28, 82]	7.1 [0.98, 19]	0.85 [0.025, 3.2]	0.078 [0.00019, 0.33]
Status Quo	Lognormal	5	52 [29, 77]	9.1 [2.3, 20]	1.6 [0.23, 4.5]	0.24 [0.017, 0.79]
Status Quo	Lognormal	10	52 [28, 77]	9 [2.4, 20]	1.6 [0.23, 4.5]	0.24 [0.017, 0.78]
Status Quo	Lognormal	20	51 [28, 77]	9 [2.3, 20]	1.6 [0.23, 4.4]	0.24 [0.017, 0.78]
Status Quo	Pareto	5	94 [84, 99]	68 [46, 87]	44 [24, 67]	25 [12, 44]
Status Quo	Pareto	10	94 [84, 99]	67 [46, 87]	44 [24, 66]	25 [11, 44]
Status Quo	Pareto	20	93 [83, 99]	67 [46, 87]	43 [24, 66]	25 [11, 44]
Status Quo	Weibull	5	49 [24, 76]	6 [0.81, 17]	0.72 [0.021, 2.6]	0.065 [0.00016, 0.28]
Status Quo	Weibull	10	48 [23, 76]	5.9 [0.81, 16]	0.71 [0.02, 2.7]	0.065 [0.00015, 0.28]
Status Quo	Weibull	20	48 [23, 76]	5.9 [0.81, 16]	0.7 [0.02, 2.6]	0.064 [0.00016, 0.28]

Projections for the cumulative (2019–2039) probability of at least one event occurring with severity for the Total Shot variable meeting each of several threshold, for models with a tail location of $x_{min}=4$. Formatting follows Table 3

Appendix D: Supplemental Event Probability Projections

See Tables 9, 10 and 11.

Table 9 Event probability projections: killed gunfire, single year, tail location of 10

Scenario	Model	Window	P_{2019} $(x > 49)$	P_{2019} $(x > 60)$	P_{2019} $(x > 75)$	P_{2019} $(x > 100)$
Optimistic	Lognormal	5	2.1 [0, 5.9]	1.4 [0, 4.2]	0.85 [0, 2.9]	0.45 [0, 1.7]
Optimistic	Lognormal	10	2.6 [0.27, 7.1]	1.7 [0.12, 5.1]	1 [0.049, 3.5]	0.56 [0.013, 2]
Optimistic	Lognormal	20	2.8 [0.33, 7.6]	1.8 [0.16, 5.4]	1.1 [0.065, 3.7]	0.61 [0.018, 2.2]
Optimistic	Pareto	5	2.6 [0, 7.1]	2 [0, 5.6]	1.5 [0, 4.3]	0.98 [0, 3.1]
Optimistic	Pareto	10	3.3 [0.44, 8.5]	2.5 [0.27, 6.7]	1.8 [0.17, 5.2]	1.2 [0.087, 3.8]
Optimistic	Pareto	20	3.5 [0.53, 9.1]	2.7 [0.34, 7.2]	2 [0.21, 5.6]	1.3 [0.11, 4]
Optimistic	Weibull	5	2 [0, 5.8]	1.3 [0, 4]	0.75 [0, 2.6]	0.36 [0, 1.4]
Optimistic	Weibull	10	2.5 [0.23, 7]	1.6 [0.096, 4.9]	0.92 [0.032, 3.2]	0.45 [0.0057, 1.7]
Optimistic	Weibull	20	2.7 [0.29, 7.5]	1.7 [0.12, 5.2]	1 [0.043, 3.4]	0.49 [0.0083, 1.9]
Pessimistic	Lognormal	5	4.4 [0.74, 11]	2.9 [0.36, 8.3]	1.8 [0.15, 5.6]	0.98 [0.045, 3.4]
Pessimistic	Lognormal	10	4.1 [0.66, 11]	2.7 [0.33, 7.7]	1.7 [0.14, 5.3]	0.91 [0.041, 3.2]
Pessimistic	Lognormal	20	3.9 [0.6, 10]	2.6 [0.3, 7.3]	1.6 [0.13, 5.1]	0.86 [0.037, 3]
Pessimistic	Pareto	5	5.6 [1.2, 13]	4.2 [0.77, 11]	3.1 [0.47, 8.4]	2.1 [0.25, 6.2]
Pessimistic	Pareto	10	5.2 [1.1, 13]	4 [0.69, 10]	2.9 [0.43, 7.9]	2 [0.23, 5.8]
Pessimistic	Pareto	20	4.9 [0.96, 12]	3.7 [0.63, 9.6]	2.8 [0.39, 7.5]	1.9 [0.21, 5.5]
Pessimistic	Weibull	5	4.3 [0.66, 11]	2.7 [0.29, 7.9]	1.6 [0.1, 5.2]	0.79 [0.022, 3]
Pessimistic	Weibull	10	4 [0.59, 10]	2.5 [0.26, 7.4]	1.5 [0.094, 4.9]	0.74 [0.02, 2.8]
Pessimistic	Weibull	20	3.8 [0.54, 10]	2.4 [0.24, 7]	1.4 [0.085, 4.7]	0.69 [0.018, 2.6]
Status Quo	Lognormal	5	3.3 [0.45, 8.7]	2.2 [0.22, 6.3]	1.3 [0.094, 4.3]	0.72 [0.027, 2.5]
Status Quo	Lognormal	10	3.3 [0.44, 8.7]	2.1 [0.22, 6.3]	1.3 [0.091, 4.3]	0.71 [0.026, 2.5]
Status Quo	Lognormal	20	3.2 [0.44, 8.7]	2.1 [0.21, 6.2]	1.3 [0.09, 4.2]	0.71 [0.026, 2.5]
Status Quo	Pareto	5	4.1 [0.72, 10]	3.1 [0.47, 8.2]	2.3 [0.29, 6.5]	1.6 [0.16, 4.7]
Status Quo	Pareto	10	4.1 [0.71, 10]	3.1 [0.46, 8.2]	2.3 [0.29, 6.4]	1.6 [0.15, 4.7]
Status Quo	Pareto	20	4.1 [0.7, 10]	3.1 [0.46, 8.2]	2.3 [0.28, 6.3]	1.5 [0.15, 4.6]
Status Quo	Weibull	5	3.1 [0.4, 8.6]	2 [0.18, 6]	1.2 [0.062, 4]	0.58 [0.013, 2.2]
Status Quo	Weibull	10	3.1 [0.39, 8.5]	2 [0.18, 6]	1.2 [0.061, 3.9]	0.58 [0.013, 2.2]
Status Quo	Weibull	20	3.1 [0.39, 8.5]	2 [0.17, 5.9]	1.2 [0.06, 3.9]	0.57 [0.012, 2.2]

Projections for the single-year (2019) probability of at least one event occurring with severity for the Killed Gunfire variable meeting each of several threshold, for models with a tail location of $x_{min} = 10$. Formatting follows Table 3

Table 10 Event probability projections: total shot, single year, tail location of 10

Scenario	Model	Window	P_{2019} ($x > 100$)	P_{2019} ($x > 250$)	P_{2019} ($x > 500$)	P_{2019} ($x > 1000$)
Optimistic	Lognormal	5	2.9 [0, 7.3]	0.43 [0, 1.3]	0.085 [0, 0.31]	0.015 [0, 0.061]
Optimistic	lognormal	10	3.6 [0.58, 8.7]	0.53 [0.044, 1.6]	0.11 [0.0042, 0.37]	0.019 [0.00028, 0.075]
Optimistic	Lognormal	20	3.9 [0.7, 9.3]	0.57 [0.055, 1.7]	0.12 [0.0054, 0.4]	0.021 [0.00037, 0.081]
Optimistic	Pareto	5	4.5 [0, 11]	1.4 [0, 3.8]	0.61 [0, 1.7]	0.26 [0, 0.8]
Optimistic	Pareto	10	5.6 [1.1, 13]	1.8 [0.25, 4.6]	0.76 [0.084, 2.1]	0.32 [0.026, 0.97]
Optimistic	Pareto	20	6.1 [1.3, 14]	1.9 [0.31, 4.9]	0.82 [0.1, 2.3]	0.35 [0.033, 1]
Optimistic	Weibull	5	2.9 [0, 7.4]	0.31 [0, 1]	0.041 [0, 0.17]	0.0042 [0, 0.019]
Optimistic	Weibull	10	3.6 [0.55, 8.8]	0.39 [0.021, 1.3]	0.051 [0.00054, 0.2]	0.0052 [0, 0.023]
Optimistic	Weibull	20	3.9 [0.66, 9.4]	0.42 [0.027, 1.4]	0.055 [0.00076, 0.22]	0.0057 [0, 0.026]
Pessimistic	Lognormal	5	6.1 [1.6, 14]	0.92 [0.13, 2.6]	0.19 [0.013, 0.63]	0.033 [0.00093, 0.13]
Pessimistic	Lognormal	10	5.7 [1.4, 13]	0.86 [0.11, 2.5]	0.17 [0.012, 0.59]	0.031 [0.00085, 0.12]
Pessimistic	Lognormal	20	5.4 [1.3, 12]	0.81 [0.1, 2.3]	0.16 [0.011, 0.55]	0.029 [0.00078, 0.11]
Pessimistic	Pareto	5	9.6 [2.9, 20]	3.1 [0.69, 7.4]	1.3 [0.23, 3.4]	0.56 [0.075, 1.6]
Pessimistic	Pareto	10	9 [2.6, 19]	2.9 [0.62, 6.9]	1.2 [0.21, 3.2]	0.52 [0.068, 1.5]
Pessimistic	Pareto	20	8.5 [2.3, 18]	2.7 [0.57, 6.6]	1.2 [0.19, 3.1]	0.49 [0.062, 1.4]
Pessimistic	Weibull	5	6.1 [1.5, 14]	0.68 [0.063, 2.1]	0.09 [0.002, 0.35]	0.0092 [1.9e-05, 0.041]
Pessimistic	Weibull	10	5.7 [1.4, 13]	0.63 [0.056, 2]	0.083 [0.0019, 0.33]	0.0085 [1.8e-05, 0.039]
Pessimistic	Weibull	20	5.4 [1.2, 12]	0.59 [0.051, 1.9]	0.079 [0.0016, 0.31]	0.0081 [1.6e-05, 0.036]
Status Quo	Lognormal	5	4.5 [0.94, 11]	0.68 [0.076, 2]	0.14 [0.0078, 0.47]	0.024 [0.00056, 0.095]
Status Quo	Lognormal	10	4.5 [0.92, 11]	0.67 [0.076, 2]	0.14 [0.0077, 0.46]	0.024 [0.00055, 0.094]
Status Quo	Lognormal	20	4.5 [0.9, 11]	0.67 [0.074, 2]	0.13 [0.0076, 0.46]	0.024 [0.00053, 0.093]
Status Quo	Pareto	5	7.1 [1.7, 16]	2.3 [0.42, 5.6]	0.97 [0.14, 2.6]	0.41 [0.045, 1.2]
Status Quo	Pareto	10	7.1 [1.7, 15]	2.3 [0.41, 5.6]	0.96 [0.14, 2.6]	0.41 [0.045, 1.2]
Status Quo	Pareto	20	7 [1.6, 15]	2.3 [0.4, 5.6]	0.95 [0.14, 2.6]	0.41 [0.045, 1.2]
Status Quo	weibull	5	4.5 [0.9, 11]	0.5 [0.038, 1.6]	0.065 [0.0012, 0.26]	0.0068 [1.1e-05, 0.03]
Status Quo	Weibull	10	4.5 [0.9, 11]	0.49 [0.038, 1.6]	0.065 [0.0012, 0.26]	0.0067 [1e-05, 0.03]
Status Quo	Weibull	20	4.5 [0.87, 11]	0.49 [0.037, 1.6]	0.064 [0.0011, 0.25]	0.0066 [1e-05, 0.03]

Projections for the single-year (2019) probability of at least one event occurring with severity for the Total Shot variable meeting each of several threshold, for models with a tail location of $x_{min} = 10$. Formatting follows Table 3

Table 11 Event probability projections: total shot, cumulative, tail location of 10

Scenario	Model	Window	$P_{2019\text{–}2039}\,(x > 100)$	$P_{2019\text{–}2039}\,(x > 250)$	$P_{2019\text{–}2039}\,(x > 500)$	$P_{2019\text{–}2039}\,(x > 1000)$
Optimistic	Lognormal	5	44 [20, 71]	8.3 [1.6, 21]	1.8 [0.16, 5.6]	0.32 [0.011, 1.2]
Optimistic	Lognormal	10	50 [24, 78]	10 [2, 26]	2.2 [0.2, 6.9]	0.39 [0.014, 1.5]
Optimistic	Lognormal	20	53 [26, 81]	11 [2.2, 27]	2.4 [0.21, 7.5]	0.43 [0.015, 1.6]
Optimistic	Pareto	5	60 [35, 83]	25 [9.1, 48]	12 [3, 26]	5.2 [0.99, 13]
Optimistic	Pareto	10	67 [42, 89]	30 [11, 55]	14 [3.7, 32]	6.4 [1.2, 16]
Optimistic	Pareto	20	70 [45, 91]	32 [12, 58]	15 [4.1, 34]	6.9 [1.4, 18]
Optimistic	Weibull	5	43 [20, 71]	6.1 [0.8, 17]	0.85 [0.025, 3.3]	0.087 [0.00022, 0.4]
Optimistic	Weibull	10	50 [24, 79]	7.5 [0.98, 21]	1 [0.031, 4]	0.11 [0.00028, 0.5]
Optimistic	Weibull	20	53 [25, 81]	8.2 [1.1, 23]	1.1 [0.033, 4.4]	0.12 [0.00031, 0.55]
Pessimistic	Lognormal	5	69 [40, 93]	17 [3.5, 40]	3.7 [0.34, 12]	0.69 [0.024, 2.6]
Pessimistic	Lognormal	10	67 [37, 92]	16 [3.3, 38]	3.5 [0.32, 11]	0.65 [0.023, 2.4]
Pessimistic	Lognormal	20	65 [35, 90]	15 [3.1, 36]	3.3 [0.3, 10]	0.61 [0.021, 2.3]
Pessimistic	Pareto	5	84 [62, 98]	45 [19, 75]	23 [6.6, 48]	11 [2.2, 27]
Pessimistic	Pareto	10	82 [59, 97]	43 [18, 73]	22 [6.2, 46]	10 [2, 25]
Pessimistic	Pareto	20	81 [57, 97]	42 [17, 71]	21 [5.8, 44]	9.6 [1.9, 24]
Pessimistic	Weibull	5	68 [38, 93]	13 [1.8, 34]	1.8 [0.055, 7]	0.19 [0.0005, 0.89]
Pessimistic	Weibull	10	66 [36, 92]	12 [1.6, 32]	1.7 [0.051, 6.6]	0.18 [0.00047, 0.82]
Pessimistic	Weibull	20	64 [34, 91]	11 [1.5, 30]	1.6 [0.048, 6.1]	0.17 [0.00043, 0.77]
Status Quo	Lognormal	5	59 [31, 86]	13 [2.6, 31]	2.8 [0.25, 8.7]	0.51 [0.017, 1.9]
Status Quo	Lognormal	10	58 [30, 86]	13 [2.5, 31]	2.8 [0.25, 8.7]	0.5 [0.017, 1.8]
Status Quo	Lognormal	20	58 [30, 85]	13 [2.5, 31]	2.7 [0.25, 8.6]	0.5 [0.017, 1.9]
Status Quo	Pareto	5	75 [50, 94]	37 [15, 64]	18 [4.9, 38]	8.1 [1.6, 20]
Status Quo	Pareto	10	75 [50, 94]	36 [14, 64]	18 [4.9, 38]	8 [1.6, 20]
Status Quo	Pareto	20	75 [50, 94]	36 [14, 64]	17 [4.8, 38]	8 [1.6, 20]
Status Quo	Weibull	5	58 [30, 86]	9.5 [1.3, 26]	1.3 [0.04, 5.2]	0.14 [0.00036, 0.64]
Status Quo	Weibull	10	58 [29, 86]	9.4 [1.3, 26]	1.3 [0.039, 5.1]	0.14 [0.00036, 0.64]
Status Quo	Weibull	20	58 [29, 86]	9.3 [1.3, 26]	1.3 [0.039, 5]	0.14 [0.00036, 0.63]

Projections for the cumulative (2019–2039) probability of at least one event occurring with severity for the Total Shot variable meeting each of several threshold, for models with a tail location of $x_{min} = 10$. Formatting follows Table 3

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